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An economic model for deriving economic values for use in selection indexes

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I. INTRODUCTION

"... As yet, neither breeders nor geneticists have developed any close collaboration with economists with a view to finding out what are the greatest economic weaknesses of the various breeds available. Selection indexes need realistic economic values not now available. The economics of breeding, including the cost of improvements and their value, ought to be known, ..." [Lerner and Donald, 28, pp. 38-39].

It is difficult for breeders, geneticists, and economists to find the "common ground" from which they can work together in formulating, researching, and solving mutual problems. Each is a specialist in his own field of study. This does not mean that it is impossible for the three areas to be incorporated, it means that only with the introduction of basic concepts from each of the areas, may the breeder, geneticist, and economist come together to formulate, research, and solve problems applicable to the three specialized fields of study.

For this thesis to be understood by specialists in the areas of genetics, animal breeding, and economics, it is imperative that basic concepts of each specialized field be presented. It is also important that some of the relationships that exist between the specialized fields of genetics, animal breeding, and economics be shown. It is for this reason that the remaining part of this introductory discussion be concentrated on certain concepts and relationships of genetics, animal breeding, and economics.

A. Genetics

For the economists, a legitimate question may be, "What is genetics?".

"Genetics is the study of hereditary potentialities, their origin, their transmission from generation to generation, and their manifestation in the life of the individual and the population" [Auerbach, 1, p. V].

Yet, what does this definition tell the economist? Hereditary potentialities is a term used by geneticists or animal breeders. To fully describe hereditary potentialities would require a detailed study of the field of genetics.

In answering the question of, "What is genetics?", a more elementary answer may be appropriate. The elementary answer can be given as: Genetics is the study of heredity and variation. A simplistic description for heredity is that it is made up of units of inheritance called genes. Genes are the basis for the genetic make-up of each separate individual whether he be man or woman, human or beast. Every individual begins life with a specific genetic make-up or array of genes which governs the individual's reactions to his environment and thereby influences the type of individual into which he develops. The differences in the specific genetic make-up of each individual cause variation within a population of individuals. Therefore, differences in heredity cause at least some of the variation within a population of individuals.

When looking at the relationships of the specialized fields of genetics and animal breeding, genetics is used as a foundation block for animal breeding. Genetics applied to individuals and populations is used as a basis from which to learn animal breeding concepts.

A first step towards understanding the theory of animal breeding is studying the behavior of genes in populations. The real basis of animal breeding is the fact that genes occur in pairs, one gene of each pair having come at random from the sire and the other from the dam. A parent transmits a sample half of his genes to each offspring. The degree of resemblance depends upon the importance of gene effects as they influence the variation of a trait. This is the basis of selection.

Another step towards understanding the theory of animal breeding through genetics is by investigating characters of an individual influenced by many pairs of genes or the genetic make-up of an individual. Another term used to describe the gene make-up of an individual is genotype. The outward expression of the individual's different characters or traits is described as the individual's phenotype. The individual's phenotype is a result of the influence of many genes and other factors. By investigating characters or traits, among related individuals, such as hair color, eye color, or feed efficiency in livestock, much can be learned about gene influence on the variation within populations.

A final step towards understanding the theory of animal breeding through genetics is by looking at non-hereditary influences on the traits. Non-hereditary influences, generally considered as environmental influences, e.g., management, climate, or nutrition, greatly influence the phenotype of an individual. The phenotype, as earlier indicated, is influenced by the genetic make-up or the genotype of the individual, but it is also influenced by nonhereditary influences or environment. The following expression defines this relationship.

1.1 $P = G + E$

where P = phenotypic value of an individual, which is measurable

G = genotypic value of an individual, which is non-measurable

E = environmental deviation

Another interpretation similar in meaning to the previous expression is that "the genotype determines the phenotypic range within which an individual will fall; the environment determines where in that range the individual will fall" [Burns, 4, p. 23].

B. Animal Breeding

In the concluding paragraph of the previous section, it was told how genetics applies to animal breeding. Yet, animal breeding has not been formally defined.

Animal breeding is the study of biological variation among domestic animals and the application of this knowledge in making genetic change. From the first part of the definition, it is seen how some knowledge of genetics is required in order to obtain an understanding of animal breeding. The second part of the definition is an applicative part of genetics used on domestic animals. Yet, how is animal improvement defined? What changes constitute domestic animal improvement?

In answering the question of what changes constitute animal improvement, there are three complex elements involved:

1) purebred breeders' incomes, unlike those of users of unregistered stock, include the sale of purebred breeding stock;

2) performance has many components, some of which may be incompatible with each other;

3) economic and environmental conditions of production are inconstant, prices and husbandry varying with time and locality [Lerner and Donald, 28, pp. 24-25]. These points are discussed in the following.

1) The role purebred breeders play is one of supplying breeding stock to the market, especially breeding stock which is purchased on the basis of appearance or type. The purebred breeders must supply the popular type of livestock in order to succeed. The purebred breeders, therefore, are entitled to their opinions as to what changes constitute improvement and that their purebred breeding stock exemplifies the improvements. Each purebred breeder, though, may consider improvement differently.

2) In the quest for animal improvement, some traits or components of improvement may not be compatible. Take, for example, cattle. Cattle have a dual purpose in providing both milk and meat. Yet, if milk production in cattle is improved, the meatiness of the animals may be reduced. In the same manner, if the meatiness of the animals is improved, the milk production of the cattle may decline. The same type of analogy can be seen in sheep. It is possible to have two types of sheep, wool producing and meat producing.

In order to produce wool and meat in large quantities, sheep are generally selected on the basis of one trait or the other, depending upon the purpose.

3) Improvers of livestock find it hard to convince others where superiority and breeding value lie. Breeding value is simply the value of an animal as a breeder or a parent. The breeding value of an individual is defined in terms of the average performance of its progeny and is a property of the individual and the population from which its mates are drawn. Numerous criteria can be found for selecting superior animals. Yet with uncertainty in the markets and the constant variation in costs of production, there seems to be much room for disagreement in the concept of the superior animal and its breeding value. Because of the disagreement and lack of direction toward the superior animal, pessimists take the attitude that improving animals is hopeless.

Changes in public taste alter demand for animal products both in quality and quantity. Technological developments alter costs of production. With these types of potential changes in mind, it is very difficult for the animal breeder to improve livestock. It is difficult for the animal breeder to know what traits to improve in his livestock so as to benefit by increased returns and profits.

Economics relates to animal breeding through technology. There are two categories of technology in animal breeding. The first category of technology applies to the overall animal industry. In this category, economics can guide the animal breeder in finding the economic importance of traits which are used as criterion in selecting animals for improvement. The second category of technology is the technology of selection schemes. In this category, economics can guide the animal breeder in evaluating the type of breeding system to use. Economics aids in evaluating selection schemes such as specialized sire and dam lines, crossing schemes, selection systems, age distributions, and the like.

Economics also relates to animal breeding through prices of inputs and outputs of production. With a certain technology available for production, prices of inputs and outputs vary with time. Here again, economics can guide the animal breeder in finding economically important traits to use as criterion in selecting animals for improvement.

C. Selection Index

One manner in which economics is used to guide the animal breeder is by finding the economic importance of traits which are used as criterion in selecting animals for improvement. The selection index is an important step toward improvement, since "genetic improvement can be induced only by selection -- by allowing genotypes (of individuals with high breeding value) to contribute to the next generation according to their relative merit" [Hazel, 14, p. 6]. The main purpose of the selection index is simultaneous selection for several traits in an effort to make maximum genetic improvement [Harris, 12, p. 3].

There are actually three kinds of purposes to which selection indexes can be put:

1) In selection for a single trait, an index incorporating information on the individual and on its various relatives, ancestors, collaterals, or descendants, increases the accuracy of estimation of the animal's genetic merit, especially for traits of low heritability (or traits that show differences between individuals or groups because of a high proportion of factors other than heredity causing the difference).

2) Selection may be directed primarily to one trait, but the index may incorporate information on other traits as an aid in identifying genetic merit.

3) As was indicated earlier, the most important use for selection indexes is in breeding populations where multiple objectives are pursued. That is, simultaneously selecting animals on the basis of several traits in an effort to make maximum genetic improvement is the selection index's primary use [Lerner and Donald, 28, pp. 85-86].

D. Construction of a Selection Index

The construction of a selection index involves the value for each of the traits used as a basis for selection. The addition of the values for each of the traits gives a total score for all of the traits, which is used as a basis for selecting the animals. The animals with the highest total scores are those which are selected. The influence of each trait on the final index is determined by the weight each trait receives in relation to the other traits. The amount of weight given to each trait depends upon its economic value, upon the heritability of each trait, and the genetic associations among the traits [Hazel and Lush, 17, p. 393].

The individual breakdown of the value for each of the traits can be shown by the aggregate breeding value. The aggregate breeding value can be shown by the following equation 1.2.

$$1.2 \qquad H = a_1G_1 + a_2G_2 + \ldots + a_mG_m = \sum_{i=1}^{m} a_iG_i$$

where H is the aggregate breeding value

a_i is the economic value (or economic weight) of the i-th trait

G_i is the breeding value of the i-th trait

m is the number of traits being considered in the selection index

$\sum_{i=1}^{m}$ indicates the summation of terms having subscripts i from 1 to and including m

The aggregate breeding value is actually a linear function defining the sum of the breeding values for a variety of traits. Aggregate breeding value is a concept and cannot be readily found for a variety of traits. The term "economic value" (or economic weight) can be defined as "... the amount by which net profit may be expected to increase for each unit of improvement in that trait" [Hazel, 15, p. 2]. The term breeding value represents "the contribution to the phenotype or observed characteristic due to all gene effects possessed by an individual" [Harris, 12, p. 4].

Because genes are particulate (individual units) and occur in pairs in animals, and because genes segregate and recombine, and because the outward expression of gene pairs is not always indicative of the gene pairs themselves, the exact genotype of traits will not be known. For this same reason, genotypic values of traits will not be known. For this same reason also, direct selection based on the aggregate breeding value is not possible. However, selection may be based upon an index, I, which is a linear function determined from the observable characteristics (phenotypes) of each of the traits and used as a basis for selection.

The index can be defined mathematically as

$$1.3 \qquad I = b_1x_1 + b_2x_2 + \ldots + b_nx_n$$

$$= \sum_{j=1}^{n} b_jx_j$$

where I is the numerical index score

b_j is the regression coefficient chosen such that r_{IH} (the correlation between the aggregate breeding value and the index) is maximized

$x_j = y_j - u_j$

where y_j is the objective phenotypic measurement of the j-th trait

u_j is a mean parameter of the phenotypic measurements of the j-th trait assumed to be known without error

x_j is the deviation from the mean parameter of the measurements of the j-th trait

n is the number of traits being considered in the selection index

$\sum_{j=1}^{n}$ indicates the summation of terms having subscript j from 1 to n, inclusive

Provided the b_j coefficients in equation 1.3 are chosen such that r_{IH} (the correlation between the aggregate breeding value and the index) is maximized, equation 1.3 has several properties:

1) It maximizes r_{IH} (also termed the accuracy of selection) as was indicated earlier.

2) It maximizes genetic progress.

3) It minimizes $E(I-H)^2$.

4) $E(H/x_1, x_2, \ldots, x_n)$ is the selection criterion in the multivariate normal case. The selection index takes as the criterion of selection the average value of the H's associated with the x_j equal to those on the individual that is a candidate for selection.

5) It maximizes the probability of selecting the better of two individuals [Henderson, 19, p. 114]

It has been shown by Smith [35] and Hazel [15] that the optimum estimates of b_j are functions of 1) the genetic and phenotypic variances and covariances of the traits in the I and H equations and 2) the economic values (or economic weights). One method for finding b_j's is using least squares. In this method, by differentiating $E(I-H)^2$ with respect to b_j, it is possible to minimize $E(I-H)^2$. Differentiating $E(I-H)^2$ yields the following set of simultaneous equations.

$$1.4 \qquad \begin{aligned} \sigma x_1x_1b_1 + \sigma x_1x_2b_2 + \ldots + \sigma x_1x_nb_n &= \sigma x_1H \\ \sigma x_2x_1b_1 + \sigma x_2x_2b_2 + \ldots + \sigma x_2x_nb_n &= \sigma x_2H \\ . \quad + \quad . \quad + \ldots + \quad . \quad &= \quad . \\ \sigma x_nx_1b_1 + \sigma x_nx_2b_2 + \ldots + \sigma x_nx_nb_n &= \sigma x_nH \end{aligned}$$

$$1.5 \qquad \begin{aligned} \sigma x_1H &= a_1\sigma g_1g_1 + a_2\sigma g_1g_2 + \ldots + a_m\sigma g_1g_m \\ \sigma x_2H &= a_1\sigma g_2g_1 + a_2\sigma g_2g_2 + \ldots + a_m\sigma g_2g_m \\ . \quad &= \quad . \quad + \quad . \quad + \ldots + \quad . \\ \sigma x_nH &= a_1\sigma g_ng_1 + a_2\sigma g_ng_2 + \ldots + a_m\sigma g_ng_m \end{aligned}$$

where σx_ix_i is the phenotypic variance of x_i with $i = 1, \ldots, n$

σx_ix_j is the phenotypic covariance of x_i and x_j with $i \neq j$ and $i = 1, \ldots, n$ and $j = 1, \ldots, n$

σg_ig_j is the genotypic covariance of g_i and g_j with $i \neq j$ and $i = 1, \ldots, n$ and $j = 1, \ldots, m$

a_i is the economic value (economic weight) of the i-th trait in the aggregate breeding value (H)

The same set of simultaneous equations will be found for maximizing r_{IH} (the correlation between the index and the aggregate breeding value) which is shown by Vandepitte [Vandepitte, 40, pp. 9-10]. These equations follow the properties of the index. That is, they maximize expected genetic progress and the probability of a correct selection.

The set of simultaneous equations may also be shown in matrix notation given the following definitions of terms:

$g = (g_1, \ldots, g_m)$ is a vector of mxl dimension; a column vector of breeding values

$a = (a_1, \ldots, a_m)$ is a vector of mxl dimension; a column vector of economic values

$p = (x_1, \ldots, x_n)$ is a vector of nxl dimension; a column vector of phenotypic measures (deviations of the measured trait from the mean parameter)

$b = (b_1, \ldots, b_n)$ is a vector of nxl dimension; a column vector of unknown weighting factors to be used in the index (actually partial regression coefficients)

P is a nxn matrix of phenotypic covariances between the n variables in p

G is a nxm matrix of genotypic covariances between the n variables in P and the m traits in H

Equation 1.2 can be written as

1.6 $\quad H = a' g$

Equation 1.3 can be written as

1.7 $\quad I = b' p$

The simultaneous equations may now be seen in matrix form. The equations, found by minimizing $E(I-H)^2$ or by maximizing r_{IH}, in matrix form, are

1.8 $\quad Pb = Ga$

From equation 1.8 it follows from elementary matrix algebra that

1.9 $$b = P^{-1} Ga$$

so, by knowing the P^{-1} and G matrices and knowing the a vector, the unknown weighting factors or partial regression coefficients may be found.

Once the partial regression coefficients are found, the index score is simple to find for each animal. Simply substitute back into equation 1.7 the newly found partial regression coefficients and the phenotypic measures, and the index score may be found.

In finding the index score for individual animals to use as a criterion for selection, it is assumed that the economic values (economic weights) of the I traits are known. It has not, however, been shown why the economic values are known or how they are derived. It is to this problem of deriving the economic values that this thesis directs its attention.

E. Objectives

The objectives of this thesis are to:

1) Formally define economic values (economic weights) used in finding the selection index score.

2) Develop an economic model which can be used to derive economic values for use in selection indexes.

3) Demonstrate the use of the economic model in deriving economic values for use in selection indexes.

The remaining parts of this thesis will include a review of literature, a discussion of methods and procedures used in the economic model for deriving the economic values, a discussion of the empirical analyses and solutions demonstrating the use of the economic model for deriving the economic values, and a discussion of summaries drawn from the empirical analyses and solutions.

The remaining thesis is quite lengthy because it presents fundamental economic concepts to animal scientists and fundamental animal breeding concepts to economists. It also integrates economic and animal breeding concepts.

II. REVIEW OF LITERATURE

As compared to the other parameters of the selection index, relatively little research has been directed toward the economic values in the selection index. The fluctuation of both prices of outputs and the cost of inputs plus the unavailability of data for certain traits has caused economic values to be approximated. Very few attempts, if any, have been made to formally define and develop a model specifically for the purpose of deriving economic values for use in selection indexes.

One of the first applications of the selection index was shown by Smith [35]. Smith developed an index to use in the selection of Australian wheat varieties. The selection of the Australian wheat varieties was based on several characteristics of the varieties. Smith assumed that the economic weights and the genetic relations for the various characters were known.

Even though the economic weights were assumed to be known in Smith's work, they were never formally defined. Smith wrote, "Suppose that in a wheat selection programme we are required to consider n characters, say x_1, x_2, ... x_n. Let us evaluate each in terms of one of them, say x_1. For example, suppose we take x_1 to represent yield of grain: x_2 may represent baking quality and we may consider that an advance of 10 in baking score is equal in value to an advance of 1 bushel per acre in yield: x_3 may represent resistance to flag smut and we may evaluate a decrease of 20 per cent infection as worth 1 bushel of yield .... Let these values be designated a_1, a_2, ..., a_n.

Then taking yield, x_1, as standard and units as indicated, we will have $a_1 = 1$, $a_2 = 0.1$, $a_3 = -0.05$, etc."

Smith, through his examples of the economic weights, showed that he viewed the economic weights as ratios. Smith never indicated how to determine whether "an advance of 10 in baking score is equal in value to an advance of 1 bushel per acre in yield" or how to determine whether "a decrease of 20 per cent infection is worth 1 bushel of yield." Thus, the problems of defining and deriving the economic weights of traits in the selection index were ignored by Smith.

One of the first applications of the selection index to animals was by Hazel [15]. Hazel developed an index to use in the selection of young boars and gilts. The selection index used 180-day weight, market score, and productivity as characters by which to base the selection of the swine. Hazel, as Smith, assumed that the economic weights were known.

In Hazel's 1943 description of the economic weights, he wrote, "The relative economic value for each trait depends upon the amount by which profit may be expected to increase for each unit of improvement in that trait. Good approximations to relative economic values often can be obtained from long-time price averages and cost-of-production figures." Hazel, in his description of the selection index, never explicitly defined relative economic value but instead related it to influencing factors. In his application of the selection index, Hazel, as Smith, used the idea that the economic weights for each of the characters should be ratios in terms of a single character.

With Hazel pioneering the use of selection indexes in animal selection, many other selection indexes have since been formed [Harris, 12, pp. 36-37]. With the formation of the many selection indexes also came the need for explicit definitions of the parameters used in the selection indexes, including an explicit definition for the economic values. Hazel [16] explicitly defined economic values and showed examples of the derivation of economic values for some characters in each of beef cattle, swine, and sheep. Hazel [16] wrote, "The economic values are of primary importance. These should reflect the net profit which will result to the livestock enterprise for one unit of change in the particular trait, but should not include the profit which might result from improvement in an associated trait." From this, it must be said that the economic value for a character should reflect the net profit expected to accrue to the livestock enterprise as the <u>direct</u> result of one unit of change in that trait. It should not include any net profit that will accrue to the livestock enterprise as the result of a change in correlated traits that may change as the initial trait changes, thereby causing net profit to accrue to the livestock enterprise indirectly.

As was indicated earlier, Hazel [16] exemplified the derivation of economic values for beef cattle, swine, and sheep. The following are excerpts from his 1956 mimeographed paper:

... The economic value of slaughter grade can be computed by the range in price between very good and very poor animals at slaughter, divided by the range in score for good and poor animals. This value should be multiplied by average selling weight. For example, if we score very good animals 9 and they sell for .20 per pound, and very poor animals 1 and they sell for .16 and average sale weight is 1000, the economic value is $\left[\frac{.20 - .16}{9-1}\right] 1000 = \left[\frac{.04}{8}\right] 1000 = \$5.00$.

One of Hazel's examples associated with swine is as follows:

... The value of growth rate is a function of labor cost, insurance, maintenance of equipment, etc. Figuring $.03 per pig per day for labor, $.002 per day for insurance, and $.003 per day for maintenance of equipment, we have $.035 per pig per day. Pigs which gained 1.6 lbs. per day instead of 1.5 would get to market 8 days sooner. On this basis, growth rate is worth 8 x $.035 = $.28 for each 1/10 lb. gain per day, or $2.80 per lb. per day ....

One of Hazel's examples associated with sheep is as follows:

... The value of a single lamb at weaning is about $11.25, while the value of twins is about $18.20. Thus, the economic value of number of lambs born is $6.95. Perhaps no additional credit should be given for triplets as mortality among them is very high ....

Comparing Hazel's 1956 examples of deriving economic values to the examples shown by Smith in 1936 and by Hazel in 1943, it can be seen that the concept of how the economic values must be represented changed substantially. The earlier work on economic values expressed the economic values as ratios. As a result of the ratio idea, the term "relative economic value" was used for economic weights used in the selection index [High, 21, p. 1].

During the 1950's and 1960's, the definition of the economic value of traits selected for using the selection index became accepted as "the amount by which profit may be expected to increase for each unit of improvement in that trait." Thus, the economic value is an absolute value instead of the relative value implied by Hazel's 1943 examples. Ironically, the definition is the exact phrase used by Hazel [15]. Hazel, though, if you remember, indicated that the economic values depended upon the change in profit, not indicating that the economic values were exactly the change in profit.

Following the formal definition as presented by Hazel [16], High [21], while constructing a selection index for beef cattle, found economic weights for a pound increase in weight and a unit increase in type score for beef cattle at weaning. The economic value for a pound increase in weight at weaning was estimated by finding the average price paid per pound for calves sold at feeder calf sales. This method was similar to examples Hazel presented in his 1956 mimeographed paper. The economic value for a unit increase in type score was estimated from the average differences in value per hundredweight between the medium, good, and choice feeder calves when they were sold at feeder calf sales. This method was similar to Hazel's example of deriving the economic value of slaughter grade presented earlier in this section.

Until the early 1970's, little attention had been given to the economic aspects of the selection index. More specifically, little work was done in improving the method of estimating economic values or studying the effect that errors of estimates of economic values had upon the selection index. Vandepitte [40] directed his attention toward the derivation of economic values and the effects of errors in the economic values as they relate to the selection index. Although Vandepitte did not evaluate the merits of possible methods of deriving economic values, he did list some possible methods that could be used [Vandepitte, 40, p. 35].

One possible method used for deriving economic values is a simple budgeting technique or what Vandepitte termed the "short cut" method. This method uses the same types of procedures shown by Hazel [16] and High [21]. By using the simple relationships of costs of inputs incurred in breeding and managing an animal and prices received in marketing an animal or its product, it is possible to estimate the economic value of a trait. By budgeting the costs and revenues of the animal and then finding the change in the costs of inputs incurred and/or price received due to a change in the trait, the net change of costs and revenues which reflect the change in profit due to a direct change in a trait can be found [Vandepitte, 40, pp. 40-46].

Another method that has been demonstrated to be useful in deriving economic weights is the multiple regression technique [Nordskog, 32, pp. 327-338]. The general problem to which the multiple regression analysis is applied is "to determine the extent to which income (y) can be predicted from different combinations of traits or performance variables (x's)." An illustration of the multiple regression equation given four variables is as follows:

2.1 $$(y-\bar{y}) = b_{yx_{1.234}} (x_1-\bar{x}_1) + b_{yx_{2.134}} (x_2-\bar{x}_2) + b_{yx_{3.124}} (x_3-\bar{x}_3) + b_{yx_{4.123}} (x_4-\bar{x}_4)$$

where the b's are partial regression coefficients.

From the partial regression coefficients ($b_{yx_{1.234}}$, $b_{yx_{2.134}}$, $b_{yx_{3.124}}$, and $b_{yx_{4.123}}$), it can be estimated that a one unit change in x_1 is worth $b_{yx_{1.234}}$ of income, a one unit change in x_2 is worth $b_{yx_{2.134}}$ of income, ... In other words, each partial regression coefficient measures the net change in income due to a change in one trait; partial regression coefficients measure the economic values.

Another possible means by which the economic values used in the selection index may be found is by iteration [Harris, 13, p. 864, and Vandepitte, 40, p. 122]. Using estimated a_i's (economic values) that are found using some method such as the "short cut" method or multiple regression analysis, an index would be constructed. (This index would be of the following expression

$$I = \sum_{j=1}^{n} b_j x_j$$

and found by the previously described method seen in chapter I.) Then by using a nonlinear aggregate breeding value equation (which may include crossproduct terms of the traits in addition to the individual trait terms) which better describes the aggregate breeding value due to possible relationships between the different traits, and through iteration of the a_i's (economic values), new estimates of a_i's are found so as to find the best linear approximation of the index equation $I = \sum_{j=1}^{n} b_j x_j$.

To compile a list of all the possible methods that may be used to derive economic value is both useless and uninformative unless a description and application of each method is given with the list.

Three possible methods that may be used in the derivation of economic values for selection indexes have been given; other methods may be available [Harris, 13, p. 864, and Vandepitte, 40, p. 35].

Much can be said for each method of estimating economic values shown in the literature. Yet, none of the methods have incorporated the use of an economic model in the estimation procedure. The most that any one method of estimating economic values has done is to use simple economic relationships of costs and returns of a single animal. The previously described methods never evaluated other possible economic interrelationships of the farm firm enterprises that could cause indirect increases in profit due to a direct change in a single trait of an animal. An example of such a case is where there is a decrease in an input needed for the feeding and marketing of an animal due to a change in a certain trait where the now in excess input may be utilized elsewhere by the farm firm to generate returns over and above its own value. Profit is increased due to less input needed for the feeding and marketing of the animal <u>and</u> due to the use of the input elsewhere by the farm firm in generating returns over and above the input's own cost.

Earlier in this chapter the economic definition of economic value was presented as "the amount by which profit may be expected to increase for each unit of improvement in the trait" [Hazel, 16]. This definition will be accepted as the definition of economic value in this thesis, with a minor change. The definition of economic value used in this thesis will be the amount by which profit of the firm may be expected to increase for each unit of improvement in a trait of a single animal.

With the revised definition of the economic value of a trait in mind, it is now possible to develop the economic model for deriving the economic values for respective traits. The basic economic model and the procedures used to derive the economic values are presented in the following chapter. Hopefully, the description of the economic model will aid the reader in seeing how the economic model differs from previously proposed models in deriving economic values for use in selection indexes.

III. METHODS AND PROCEDURES

The purpose of this study is to use linear programming to derive economic values for use in selection indexes. It is by the use of linear programming that an economic model is developed in order to derive economic values. This, though, will only become evident upon looking at linear programming and linear programming theory of a profit maximizing firm more carefully.

A. Fundamental Concepts and Assumptions

Prior to any discussion of the selection index and economic values, it was necessary that fundamental concepts of genetics and animal breeding be revealed. In the same manner, prior to any discussion of the proposed economic model for deriving economic values, it is necessary that fundamental concepts of linear programming be revealed.

1. Fundamental concepts of linear programming

A fundamental concept in linear programming is the "activity." The term "activity" is more or less synonymous with process, except that activity may be used in a somewhat broader context. More specifically, activity means a way of producing something by a firm (or farm). (A firm being any technical unit in which output is produced.) Thus, if a farm produced market hogs by two different techniques, these two different techniques would be considered to represent two different activities. Activities are the alternative ways in which to produce different types of output, or, in some cases the same output.

A second fundamental concept in linear programming is the concept of "inputs." An "input" may be defined as "any good or service which contributes to the production of an output" [Henderson and Quandt, 20, p. 53]. A firm will normally use many different inputs for the production of an output. It is possible that some of the inputs used in one firm may be outputs of other firms.

Inputs are classified as "fixed" or "variable" with respect to their availability in the production of outputs. The distinction between fixed and variable inputs, though, is temporal. Inputs that are classified as fixed for one period of time are actually variable for a longer period of time.

A "fixed input" is defined as an input that is necessary for the production of output, but where the quantity available for the production of output is limited or "fixed." A "variable input" is defined as an input that is necessary for the production of output, but where the quantity available for the production of output is unlimited or "variable."

As a result of classifying inputs as "fixed" or "variable," total costs can be classified as "fixed" or "variable." "Costs" are another fundamental concept in linear programming. Total cost is defined as the cost of production which results from using fixed and variable inputs in the production of output. "Fixed" cost is defined as the cost of fixed inputs. "Variable" cost is defined as the cost of variable inputs.

Another fundamental concept in linear programming is the concept of the "objective function." The "objective function," sometimes called the criterion function, defines the goal or objective of the linear program. It is the objective function which is optimized when solving the linear programming problem.

It is possible to optimize the objective function by either maximization or minimization, depending upon the objective. Maximization of the objective function is often used when the objective function expresses the returns of various "activities" of the linear programming problem and when the objective is to maximize profits. Minimization of the objective function is often used when the objective function expresses the costs of various "activities" of the linear programming problem and when the objective is to minimize costs.

By using these concepts, linear programming can be used to develop an economic theory of a competitive profit-maximizing firm. The firm has a set of fixed inputs available for use. The firm owns, for example, a certain number of machines; the firm has available a certain number of buildings; the firm has available certain amounts of natural resources, etc. The firm uses these fixed inputs together with variable inputs to produce one or more different types of output. The firm purchases each unit of variable input it needs at a constant price. The firm sells each unit of output also at a constant price. Thus, the firm faces the problem of determining the amount of variable inputs to purchase and combine with its fixed inputs, while also determining the quantities of outputs to produce, in order to maximize its profit.

2. Fundamental assumptions of linear programming

a. Additivity - linearity   Additivity - linearity means that the activities of the linear programming problem must be additive in that when two or more activities are used to produce a type of output, the total amount of output must be the sum of each individual activity's output. An equivalent statement is: the total amount of inputs used by several activities must be equal to the sum of the inputs used by each individual activity.

From this it can be seen that no interaction is possible in the amount of inputs required per unit of output regardless of whether activities are undertaken alone or in various proportions. Varying an activity by some proportion is only accomplished by varying the amount of all inputs used in the activity by that same proportion. Also, two or more activities can be carried on simultaneously, yet independently of each other. If this happens, though, the inputs required per unit of output of each activity are the same as the inputs required per unit of output of each activity that would exist if only one activity were carried out.

The idea of the inputs per unit of output being proportional to the level of output and the idea that two or more activities can be carried on simultaneously, yet independently of each other, result in linearity. With output and inputs per unit of output being additive, it must be also said that they are additive in the sense that they are linearly combined.

b. Divisibility   The divisibility assumption means that inputs can be used and output produced in quantities that can be fractional.

This means that inputs and outputs are considered to be continuous or infinitely divisible. This assumption is not as serious as it may seem since rounding quantities of output to the nearest whole unit does not cause serious decision-making errors [Heady and Candler, 18, p. 18].

c. Finiteness The finiteness assumption means that there is a limit to the number of alternative activities used to produce output and there is a limit to the input restrictions which need to be considered.

d. Single-value expectations The single-value expectations assumption means that input availability, inputs needed per unit of output, and prices are known with certainty or based on certainty equivalents. This assumption is not as serious as it may seem, since this self-same assumption is used by other research techniques such as budgeting.

B. The General Linear Programming Model

In the previous two sections fundamental concepts and assumptions of linear programming were introduced. By understanding the fundamental concepts and assumptions of linear programming, it is easy to understand the mathematical expressions of the linear program.

The typical linear program is mathematically expressed as the following:

(1) Maximize (or minimize) $\sum_{j=1}^{n} c_j x_j$

$$(2)\quad \text{subject to} \quad \sum_{j=1}^{n} a_{ij}x_j \leq a_{io};\ i = 1, 2, \ldots, m_o$$

$$(3)\quad \sum_{j=1}^{n} a_{ij}x_j \geq a_{io};\ i = m_o + 1, m_o + 2, \ldots, m_1$$

$$(4)\quad \sum_{j=1}^{n} a_{ij}x_j = a_{io};\ i = m_1 + 1, m_1 + 2, \ldots, m$$

$$(5)\quad x_1, x_2, x_3, \ldots x_n \geq 0$$

where equation (1) is the objective function (or the profit function in the case of a profit-maximizing firm).

expressions (2) - (4) are constraints (or the possible relationships of fixed input availabilities to input use in the case of a profit-maximizing firm).

inequation (2) being a "less than or equal to" constraint (where the amount of fixed input used ($\sum_{j=1}^{n} a_{ij}x_j$) must be less than or equal to the fixed input available (a_{io})).

inequation (3) being a "greater than or equal to" constraint (where the amount of fixed input used ($\sum_{j=1}^{n} a_{ij}x_j$) must be greater than or equal to the fixed input available (a_{io})).

equation (4) being an "equality" constraint (where the amount of fixed input used ($\sum_{j=1}^{n} a_{ij}x_j$) must equal the fixed input available (a_{io})).

inequation (5) is the non-negativity constraint (or the constraint that indicates no negative quantities can be produced in the respective activities).

C. An Example Model of Linear Programming Theory of the Firm

In this example it is assumed that there is a firm that feeds cattle to slaughter weight and then markets the cattle. The firm has three alternative processes which it may use to finish cattle to slaughter weight. The first process is to buy feeder calves, feed them a high roughage ration, and then sell them for slaughter. The second process is to buy feeder calves, feed them a high grain ration, and then sell them for slaughter. The third process is to buy yearling steers, feed them a medium roughage-medium grain ration, and then sell them for slaughter.

The firm has a set of fixed inputs available for use in feeding the cattle. The firm has 11,000 bushels of corn, 900 tons of silage, 300 tons of hay, and 1600 hours of labor. The feed inputs are fixed in availability because they equal the amounts of feeds the firm has produced and the firm is unwilling to sell or buy any of these feeds. The time input is fixed in availability because it is the maximum amount of time the firm feels it can allot to the processes of finishing the cattle to slaughter weight.

The firm also requires a set of variable inputs for use in feeding the cattle. The firm requires such things as supplement, veterinary services and medicine, machinery and equipment and power and fuel, and other miscellaneous variable inputs. These inputs are variable in availability because they are available in unlimited quantities and may be found in many different places with no limit on availability. These inputs, if purchased, are also available at a constant price.

The firm's objective is to maximize profits of finishing cattle to slaughter weight. The firm thus faces the problem of determining the amount of variable inputs to purchase and use with the fixed inputs, while also determining the number of cattle to finish under each process in order to maximize its profits.

Before the linear programming model is set up to find the optimum, additional data is needed. In addition to alternative processes of production (also termed "activities" and shown as x_j in the mathematical linear programming model), the levels of fixed inputs (shown as a_{io} in the mathematical linear programming model), the needed variable inputs, and the firm's objective which have already been defined, there are two types of production coefficients that must be defined. The first type of production coefficient is the production coefficient of fixed inputs (shown as a_{ij} in the mathematical linear programming model). This type of production coefficient provides information to the model about the amount of fixed input i it takes to produce one unit of output under production activity j.

The production coefficients of fixed inputs in this example are 40 bushels of corn, 3.25 tons of silage, 0.11 tons of hay, and 6.0 hours of labor needed to finish one steer to slaughter weight, fed the high roughage ration. The other production coefficients of fixed inputs are 50 bushels of corn, 0.72 tons of silage, 0.25 tons of hay, and 6.4 hours of labor needed to finish one steer to slaughter weight, fed the high grain ration, and 35 bushels of corn, 2.0 tons of silage, 0.3 tons of hay, and 5.0 hours of labor needed to finish one yearling steer to slaughter weight, fed the medium roughage-medium grain ration.

By using the production coefficients of fixed inputs along with the levels of fixed inputs, the constraint expressions of the linear program can be formed. The constraint expressions can be shown as the following:

3.1 (1) $40\ x_1 + 50\ x_2 + 35\ x_3 \leq 11{,}000$ (corn)

(2) $3.25\ x_1 + 0.72\ x_2 + 2.0\ x_3 \leq 900$ (silage)

(3) $.11\ x_1 + 0.25\ x_2 + 0.3\ x_3 \leq 300$ (hay)

(4) $6.0\ x_1 + 6.4\ x_2 + 5.0\ x_3 \leq 1600$ (labor)

(5) $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

These expressions tell us, taking expression 3.1 (1) for example, that the 40 bushels of corn needed to finish out a steer to market weight in activity #1 times the number of steers finished out to market weight in activity #1 (x_1) plus the 50 bushels of corn needed to finish out a steer to market weight in activity #2 times the number of steers finished out to market weight in activity #2 (x_2) plus the 35 bushels of corn needed to finish out a steer to market weight in activity #3 times the number of steers finished out to market weight in activity #3 (x_3) must be less than or equal to the 11,000 bushels of corn that is available at a fixed level. Taking inequation 3.1 (2) for example, the 3.25 tons of silage needed to finish out a steer to market weight in activity #1 times the number of steers finished out to market weight in activity #1 (x_1) plus the 0.72 tons of silage needed to finish out a steer to market weight in activity #2 times the number of steers finished out to market weight in activity #2 (x_2) plus the 2.0 tons of silage needed to finish out a steer to market weight in activity #3 times the number of steers finished out to market weight in activity #3 (x_3) must be less than or equal to the 900 tons of silage that is available at a fixed level. Inequations 3.1 (3) and 3.1 (4) may also be interpreted in a similar manner for hay and hours of labor, respectively. Inequation 3.1 (5) is a set of non-negativity constraints such that the number of steers finished out to market weight in activities #1, #2, and #3 cannot be negative numbers.

The second type of production coefficient is the production coefficient of variable inputs (shown as q_{kj} in the mathematical formulation of the objective function to be shown later). This type of coefficient provides information to the model about the amount of variable input, k, it takes to produce one unit of output under production activity j. Some of the possible production coefficients of variable inputs are 0.125 tons of supplement and 5 gallons of gasoline to produce a steer by a certain activity.

The final data requirements needed are the price expectations of both inputs and outputs and the objective function. The price expectations are used to form the objective function. This is because the objective function consists of the production activities (x_j's) and coefficients that describe the net return of selling one unit of output produced by each production activity. If there are J possible activities to produce the firm's output, in order to form the J different c_j coefficients of the objective function, the following equation may be used for each c_j.

3.2 $$p_j - \sum_k r_k q_{kj} = c_j$$

where p_j is the price received for one unit of output produced by the j-th activity

r_k is the purchase price of the k-th variable input

q_{kj} is the production coefficient of variable inputs which gives the quantity used of the k-th variable input in the production of one unit of output under the j-th activity

c_j is the net revenue received by producing and selling one unit of output under the j-th activity

(Note: In the case where the production activity includes no selling of the output, p_j equals zero and c_j becomes negative.)

Another equation that may be used to compute the c_j coefficients of the objective function is as follows:

3.3 $$p_j - \sum_k V_{kj} = c_j$$

where $V_{kj} = r_k q_{kj}$

and where the new variable, V_{kj}, is defined as the per unit cost of the k-th variable input and where $\sum_k V_{kj}$ is the total cost of all variable inputs used in producing one unit of output by the j-th activity.

Assume the price expectations to be:

Purchasing choice 450# calves	--	\$44.50/cwt.
Purchasing choice 650# yearlings	--	\$40.50/cwt.
Marketing choice 1050# steers	--	\$35.00/cwt.
Marketing choice 1100# steers	--	\$36.00/cwt.

Supplement costs are:

for high roughage ration	--	\$24.00/600# gain
for high grain ration	--	\$28.44/600# gain
for medium roughage-medium grain ration	--	\$14.40/350# gain

Veterinary services and medical costs are:

for steer on high roughage ration	--	\$ 9.40/steer
for steer on high grain ration	--	\$12.50/steer
for yearling steer	--	\$ 4.50/steer

Machinery and equipment and power and fuel costs are:

for steer on high roughage ration	--	\$ 9.50/steer
for steer on high grain ration	--	\$12.00/steer
for yearling steer	--	\$ 7.22/steer

Miscellaneous costs are:

for steer on high roughage ration	--	\$ 1.00/steer
for steer on high grain ration	--	\$ 1.50/steer
for yearling steer	--	\$ .75/steer

(Note: Supplement, veterinary services and medical costs, machinery and equipment and power and fuel costs, and miscellaneous costs are shown as costs of the k-th variable input (V_{kj} from equation 3.3). Each of these costs may be broken down into the purchase price of the k-th variable input, r_k, and the production coefficient of variable inputs, q_{kj}, if necessary. They will not be broken down into these variables in this example.)

It is known from equation 3.3 that

$$p_j - \sum_k V_{kj} = c_j$$

Thus, to find the net revenue received by producing and selling one unit of output under each activity, it is necessary to substitute the price expectations into equation 3.3.

For activity #1

Selling the steer at 1050# at \$35.00/cwt. = \$367.50 minus the variable input costs:

supplement	24.00	
veterinary and medical	9.40	
machinery and equipment and power and fuel	9.50	
miscellaneous	1.00	
feeder calf (which weighs 450# and is purchased at \$44.50/cwt.	200.25	
		\$244.15
gives the net revenue for activity #1 or (c_1)		\$123.35

(Note: The purchase price of the steer at \$44.50/cwt. is an example of a purchase price of the k-th variable input, r_k, and the steer weighing 450# is an example of a production coefficient of variable input, q_{kj}, as was shown in equation 3.2. The total purchase price of the steer, \$200.25, is an example of the cost of the k-th variable input, V_{kj}, shown in equation 3.3. The summation of the variable input costs is an example of $\sum_k V_{kj}$ or the total cost of all variable inputs used in producing one unit of output by the j-th activity.)

Following the same procedure for the other two activities, it can be found that the c_j for activity #2 equals \$112.66 and the c_j for activity #3 equals \$105.88.

The objective function is:

3.4 $$123.35\ x_1 + 112.66\ x_2 + 105.88\ x_3$$

This is formed by using the c_j's found previously for each activity and multiplying each c_j times the respective activity variable x_j.

Since the objective of the firm is to maximize its profits and each c_j represents the net revenue of each respective activity, the maximized objective function will give the maximum profit of the firm. The profit of the firm will be maximized provided that the firm produces the level of output in each activity as indicated in the optimal solution.

Combining the objective function and the constraint equations, in this profit maximizing problem, produces the linear program

3.4 $$\text{MAX } 123.35\ x_1 + 112.66\ x_2 + 105.88\ x_3$$

subject to the constraints

3.1 (1) $40\ x_1 + 50\ x_2 + 35\ x_3 \leq 11{,}000$

(2) $3.25\ x_1 + 0.72\ x_2 + 2.0\ x_3 \leq 900$

(3) $0.11\ x_1 + 0.25\ x_2 + 0.3\ x_3 \leq 300$

(4) $6.0\ x_1 + 6.4\ x_2 + 5.0\ x_3 \leq 1600$

(5) $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

By solving the problem, it is possible to implicitly find the amounts of variable inputs to purchase (through $q_{kj}\ x_j$ where q_{kj} is used in equation 3.2 and x_j is the variable that is solved for in the linear program), explicitly find the quantities of fixed inputs used with the variable inputs, explicitly find the quantity of output produced in each activity (x_j), and explicitly find the maximized profit of the firm. In other words, by solving the linear program, it is possible to solve the problem of the profit maximizing firm.

D. Maximization Problem

Having looked at a linear programming theory of the firm, it is now appropriate to look at linear programming in a little more depth. Linear programming is actually a mathematical technique used to solve problems. Both maximization and minimization type problems can be solved using linear programming, as was alluded to earlier. Linear programming being a mathematical technique, the following will be a mathematical presentation.

The following will be a mathematical presentation of the maximization linear program. This is because of the importance that maximization plays in finding economic values for use in selection indexes. Maximization is important in finding economic values because of its relationship with profit maximization in the theory of the firm. Economic values are changes in profits.

1. Typical maximization linear program

A typical maximization linear program can be written as

3.5 Maximize $$Z = \sum_{j=1}^{n} c_j x_j$$

3.6 subject to (1) $$\sum_{j=1}^{n} a_{ij} x_j \leq a_{io};\ i = 1, 2, \ldots, m_o$$

(2) $$\sum_{j=1}^{n} a_{ij} x_j \geq a_{io};\ i = m_o + 1, m_o + 2, \ldots, m_1$$

(3) $$\sum_{j=1}^{n} a_{ij} x_j = a_{io};\ i = m_1 + 1, m_1 + 2, \ldots, m$$

where all $x_j \geq 0$

This was seen earlier as the linear programming theory of the firm was presented. Again, the objective of the maximization linear program is to maximize. Equation 3.5 is the objective function which is maximized. The type of constraint normally associated with a maximization linear program is shown by inequation 3.6 (1). This constraint is a less than or equal to constraint. Other possible types of constraints which are associated with a maximization linear program are shown by inequation 3.6 (2) and equation 3.6 (3). These constraints are greater than or equal to and equality constraints shown in inequation 3.6 (2) and equation 3.6 (3), respectively.

The data requirements are again the same for this typical maximization linear program. c_j, a_{ij}, and a_{io} are all parameters which must be defined with respect to the values they carry in order to solve the problem. The c_j's are objective function coefficients. The a_{ij}'s are sometimes termed input-output coefficients in that they defined the amount of input needed to produce a unit of output. The a_{io}'s are sometimes termed right hand side (RHS) coefficients and constrain the total amount of inputs used.

The x_j variables are termed "real variables" in the typical maximization linear program. These variables, again, are variables which are explicitly solved for in the linear program and represent the optimal quantities of each of the activities. The x_j variables are termed "real" variables in order to differentiate them from the "slack" variables which are necessary to solve the linear program.

2. Solution procedure of a maximization linear program

Before solving a maximization linear program, it is necessary to alter the constraints (shown in inequations 3.6 (1) and (2), and equation (3)) slightly. It is necessary to convert the inequality constraints (shown in inequations 3.6 (1) and 3.6 (2)) to equality constraints by adding a non-negative "slack" variable to the less than or equal to constraint shown in inequation 3.6 (1) and by subtracting a non-negative "slack" variable from the greater than or equal to constraint shown in inequation 3.6 (2). The "slack" variables affect the constraints and have no effect on the objective function. This can be seen in the following revised maximization linear program with equality constraints developed from the less than or equal to and the greater than or equal to constraints by adding "slack" variables.

3.7 Maximize $Z = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m_o} 0x_{n+i} + \sum_{i=m_2+1}^{m_1} 0x_{n+i}$

3.8 subject to

(1) $\sum_{j=1}^{n} a_{ij}x_j + x_{n+i} = a_{io}$ where $i = 1, 2, \ldots, m_o$

(2) $\sum_{j=1}^{n} a_{ij}x_j - x_{n+i} = a_{io}$ where $i = m_o+1, m_o+2, \ldots, m_1$

(3) $\sum_{j=1}^{n} a_{ij}x_j = a_{io}$ where $i = m_1+1, m_1+2, \ldots, m$

where all "real" variables $(x_j) \geq 0$ and

all "slack" variables $(x_{n+i}) \geq 0$

Equation 3.8 may also be shown in another form. By defining A_{j_2} equal to the column vector multiplying each x_j (real variable) of the less than or equal to constraint, or inequation 3.6 (1), by defining A_{j_1} equal to the column vector multiplying each x_j (real variable) of the greater than or equal to constraint, or inequation 3.6 (2), and by defining A_{j_o} equal to the column vector multiplying each x_j (real variable) of the equality constraint, or equation 3.6 (3),

$$A_{j_2} = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{m_o j} \end{pmatrix} \quad A_{j_1} = \begin{pmatrix} a_{(m_o+1)j} \\ a_{(m_o+2)j} \\ \vdots \\ a_{m_1 j} \end{pmatrix} \quad A_{j_o} = \begin{pmatrix} a_{(m_1+1)j} \\ a_{(m_1+2)j} \\ \vdots \\ a_{mj} \end{pmatrix}$$

and by defining I as the identity matrix which has 1's running down the diagonal from upper left to lower right with 0's everywhere else

$$I = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

and by defining X_{s_1} and X_{s_2} as column vectors of x_{n+i} (slack) variables with X_{s_1} consisting of slack variables of [the less than or equal to constraint, $(i = 1, 2, \dots, m_o)$] equation 3.8 (1), and with X_{s_2} consisting of slack variables of [the greater than or equal to constraint, $(i = m_o+1, m_o+2, \dots, m_1)$] equation 3.8 (2),

$$X_{s_1} = \begin{pmatrix} x_{n+1} \\ x_{n+2} \\ x_{n+3} \\ \vdots \\ x_{n+m_o} \end{pmatrix} \qquad X_{s_2} = \begin{pmatrix} x_{n+(m_o+1)} \\ x_{n+(m_o+2)} \\ x_{n+(m_o+3)} \\ \vdots \\ x_{n+m_1} \end{pmatrix}$$

and by defining A_{o_2} equal to the column vector of right hand side coefficients for the less than or equal to constraint, or inequation 3.6 (1), by defining A_{o_1} equal to the column vector of right hand side coefficients for the greater than or equal to constraint, or inequation 3.6 (2), and by defining A_{o_o} equal to the column vector of right hand side coefficients for the equality constraint, or equation 3.6 (3),

$$A_{o_2} = \begin{pmatrix} a_{1o} \\ a_{2o} \\ \vdots \\ a_{m_o o} \end{pmatrix} \qquad A_{o_1} = \begin{pmatrix} a_{(m_o+1)o} \\ a_{(m_o+2)_o} \\ \vdots \\ a_{m_1 o} \end{pmatrix} \qquad A_{o_o} = \begin{pmatrix} a_{(m_1+1)o} \\ a_{(m_1+2)o} \\ \vdots \\ a_{mo} \end{pmatrix}$$

equation 3.8 may be written as

3.9 (1) $\sum_{j=1}^{n} A_{j_2} x_j + I_{m_o} X_{s_1} = A_{o_2}$

(2) $\sum_{j=1}^{n} A_{j_1} x_j - I_{m_1} X_{s_2} = A_{o_1}$

(3) $\sum_{j=1}^{n} A_{j_o} x_j = A_{o_o}$

where A_{j_2} is an m_ox1 column vector

A_{j_1} is an (m_1-m_o)x1 column vector

A_{j_o} is an $(m-m_1)$x1 column vector

I_{m_o} is an m_o x m_o identity matrix

I_{m_1} is an (m_1-m_o) x (m_1-m_o) identity matrix

X_{s_1} is an m_o x 1 column vector

X_{s_2} is an (m_1-m_o) column vector

A_{o_2} is an m_o x 1 column vector

A_{o_1} is an (m_1-m_o)x1 column vector

and A_{o_o} is an $(m-m_1)$x1 column vector

Using matrix algebra, equations 3.9 (1), (2), and (3) can be stated in a single equation

$$3.10 \qquad \sum_{j=1}^{n} A_j x_j + \begin{pmatrix} I_{m_o} & O_m \\ O_m & -I_{m_1} \\ O_m & O_m \end{pmatrix} \begin{pmatrix} X_{s_1} \\ X_{s_2} \end{pmatrix} = A_o$$

$$\text{where } A_j = \begin{pmatrix} A_{j_2} \\ A_{j_1} \\ A_{j_o} \end{pmatrix}$$

and is a column vector of mxl dimension

O_m is a submatrix of zeroes

$$A_o = \begin{pmatrix} A_{o_2} \\ A_{o_1} \\ A_{o_o} \end{pmatrix}$$

and is a column vector of mxl dimension

and $\begin{pmatrix} I_{m_o} & O_m \\ O_m & -I_{m_1} \\ O_m & O_m \end{pmatrix}$ is a matrix of mxm_1 dimension

With the constraints shown as equalities, due to the addition of slack variables, it is still impossible to solve the linear program. It is necessary to have an mxm identity matrix totally developed within the constraints in order to solve the linear program [Heady and Candler, 18, pp. 116-121]. The need of the identity matrix will become clear in the discussion of the method of finding the solution.

Looking at equation 3.10, it can be seen that only a partial identity matrix is developed with the constraints. Thus, it is necessary to add another set of variables to the constraints so as to totally develop the identity matrix within the constraints so as to solve the linear program. "Artificial" variables are variables that are added to the constraints so as to develop the identity matrix within the constraints. The variables are termed artificial because they actually have no meaning for the original set of constraints. "Artificial" variables, unlike "slack" variables, affect both the objective function and the constraints.

"Artificial" variables are non-negative variables when added to the constraints. Again, they allow the identity matrix to be formed within the constraints. The "artificial" variables carry a highly negative coefficient in the objective function in the case of maximization so as not to enter into the solution. They must not enter into solution because they carry no meaning other than allowing the identity matrix to be formed within the constraint.

By adding artificial variables to equation 3.10 and recombining terms, the new constraint can be written

$$3.11 \qquad \sum_{j=1}^{n} A_j x_j + \begin{pmatrix} I_{m_o} & O_m & O_m \\ O_m & I_{m_1} & O_m \\ O_m & O_m & I_{m_2} \end{pmatrix} \begin{pmatrix} X_{s_1} \\ X_{a_1} \\ X_{a_2} \end{pmatrix} - \begin{pmatrix} 0 \\ I_{m_1} \\ 0 \end{pmatrix} X_{s_2} = A_o$$

where X_{a_1} is a column subvector of artificial variables of $(m_1-m_o)x1$ dimension such that

$$X_{a_1} = \begin{pmatrix} x_{n+(m+m_o+1)} \\ x_{n+(m+m_o+2)} \\ \vdots \\ x_{n+(m+m_1)} \end{pmatrix}$$

and X_{a_2} is a column subvector of artificial variables of $(m-m_1)x1$ dimension such that

$$X_{a_2} = \begin{pmatrix} x_{n+(m_1+1)} \\ x_{n+(m_1+2)} \\ \vdots \\ x_{n+m} \end{pmatrix}$$

It was indicated earlier that the artificial variables affect the objective function in addition to the constraints. It was also indicated that the artificial variables carry highly negative coefficients in the objective function so as not to enter into the solution. This can be seen in the following equation. By changing equation 3.7 to notation consistent with equation 3.11 and by adding artificial variables, the following equation results.

3.12 Maximize $Z = \sum_{j=1}^{n} c_j\ x_j + O_{v_1} X_{s_1} + M_1 X_{a_1} +$

$$M_2 X_{a_2} + O_{v_2} X_{s_2}$$

where O_{v_1} is a row vector of zeroes of $1xm_o$ dimension

M_1 is a row vector of highly negative numbers of $1x(m_1-m_o)$ dimension

M_2 is a row vector of highly negative numbers of $1x(m-m_1)$ dimension

O_{v_2} is a row vector of zeroes of $1x(m_1-m_o)$ dimension.

Thus, in solving the maximization linear program, the revised maximization linear program is written

$$3.12 \qquad \text{Maximize } Z = \sum_{j=1}^{n} c_j \, x_j + 0_{v_1} X_{s_1} + M_1 X_{a_1} + M_2 X_{a_2} + 0_{v_2} X_{s_2}$$

subject to

$$3.11\ (a) \qquad \sum_{j=1}^{n} A_j \, x_j + I_m \begin{pmatrix} X_{s_1} \\ X_{a_1} \\ X_{a_2} \end{pmatrix} - J_M X_{s_2} = A_o$$

where I_m is the identity matrix

$$I_m = \begin{pmatrix} I_{m_o} & 0_m & 0_m \\ 0_m & I_{m_1} & 0_m \\ 0_m & 0_m & I_{m_2} \end{pmatrix}$$

of mxm dimension

and J_M is a matrix

$$J_M = \begin{pmatrix} 0_m \\ I_{m_1} \\ 0_m \end{pmatrix}$$

of $mx(m_1 - m_o)$ dimension

Various types of solutions may be found in solving a linear program. A <u>solution</u> to the linear program shown in equation 3.5 and expressions 3.6 is a vector of x's of n dimension which would satisfy expressions 3.6 or equation 3.11 (a). A <u>feasible</u> <u>solution</u> is a solution which satisfies the inequality constraints shown in expressions 3.6 or the equality constraints shown in equation 3.11 (a), but where the solution also satisfies the condition that all real variables (x_j) are greater than or equal to zero (or non-negative). A basic feasible solution to the linear program shown in equation 3.5 and expressions 3.6 is a feasible solution that contains m variables, and the m vectors (A_j) that are multiplied by these m variables in the solution are linearly independent, and all other variables (there will be n-m variables left) are zero. For example, when $x_1 = x_1^1 > 0$, $x_2 = x_2^1 > 0$, ..., $x_m = x_m^1 > 0$, $x_{m+1} = x_{m+2} = x_{m+3} = \ldots = x_{m+n} = 0$ and A_1, A_2, ..., A_m are linearly independent, this is a basic feasible solution, since the vectors A_1, A_2, A_3, ..., A_m form a basis in m-space and the matrix formed by the vectors A_1, A_2, ..., A_m is non-singular (or in other words, it has an inverse) [Ladd, 25, p. 6-2]. An optimal feasible solution is a feasible solution that maximizes the objective function or the value of Z.

In solving a maximization linear program, an optimal feasible solution is found. A fundamental linear programming theorem says: If a linear program has an optimal feasible solution, it has a basic optimal feasible solution. Because of this theorem, in solving a linear program, it is only necessary to investigate basic feasible solutions to the linear program in finding the optimal feasible solution.

A method termed the simplex method can be used to investigate basic feasible solutions to the linear program in finding the optimal feasible solution. Consider the maximization linear program shown in equations 3.12 and 3.11 (a) which can also be written as

3.13 Maximize $(C, O_v, M) \begin{pmatrix} X \\ X_s \\ X_a \end{pmatrix}$

3.14 subject to

$$(A, -J_M, I_m) \begin{pmatrix} X \\ X_s \\ X_a \end{pmatrix} = A_o$$

where (C, O_v, M) is a row vector composed of the row vectors C, O_v, and M and is of 1 x $(n+m+(m_1-m_o))$ dimension, and

where C is a row vector of c_j with $j = 1, 2, \ldots, n$, and is of 1xn dimension

O_v is a row vector of zeroes composed of O_{v_1} and O_{v_2} and is of $1xm_1$ dimension

M is a row vector of highly negative numbers composed of M_1 and M_2 and is of $1x(m-m_o)$ dimension

$\begin{pmatrix} X \\ X_s \\ X_a \end{pmatrix}$ is a column vector composed of the column vectors X, X_s, and X_a and is of $(n+m+(m_1-m_o))x1$ dimension and where X is a column vector of x_j with $j = 1, 2, \ldots, n$, and is of $n \times 1$ dimension

X_s is a column vector of x_{n+i} (slack variables) composed of X_{s_1} and X_{s_2} and is of $m_1 \times 1$ dimension

X_a is a column vector of x_{n+i} (artificial variables) composed of X_{a_1} and X_{a_2} and is of $(m-m_o) \times 1$ dimension

$(A_1 - J_M, I_m)$ is a matrix composed of the matrices $A_1 - J_M$, and I_m and is of $m \times (n+m+(m_1-m_o))$ dimension and

where A is a matrix of a_{ij} developed from column vectors of A; where $j = 1, 2, \ldots, n$

I_m, $- J_M$ and A_o are defined as in equations 3.10 and 3.11 (a)

Now, when any basis solution is selected, equation 3.11 can be written

$$BX_B + NX_N = A_o \tag{3.15}$$

where B is an $m \times m$ matrix formed by m linearly independent vectors that form a basis in m-space

X_B is an mx1 vector of values of the basic variables (or those variables that are in the basic solution)

N is an mx(n-m) matrix formed by n-m vectors that are not linearly independent vectors that form the basis in m-space

X_N is an (n-m)x1 vector of values (of zero) of the non-basic variables (or those variables that are not in the basic solution)

Since X_N is a vector composed of zero values, it follows that

3.16 $$BX_B = A_o$$

and thereby

3.17 $$X_B = B^{-1} A_o$$

If c'_B is defined as the vector of weights, from the objective function (shown in equation 3.13) of the basic variables, then the value of the objective function will be

3.18 $$Z_B = c'_B X_B$$

$$= c'_B B^{-1} A_o$$

For $j \leq n$, A_j was defined immediately after equation 3.10. Now, for $j > n$, define A_j as a column in I_m or J_M defined immediately following equation 3.11 (a). For example, A_{n+1} is the first column of I_m or

$$A_{n+1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Now, X_j can be defined as

3.19 $$X_j = B^{-1} A_j$$

for all j.

Also, for every A_j, where the j-th vector multiplies real, slack, or artificial variables, define a z_j such that

3.20 $$z_j = c'_B B^{-1} A_j$$

$$= c'_B X_j$$

Given that each variable x_j, where the j-th variable is a real, slack, or artificial variable, has a c_j associated with it in the objective function, it follows from equation 3.20 that

3.21 $$z_j - c_j = c'_B B^{-1} A_j - c_j$$

$$= c'_B X_j - c_j$$

Using the material presented above, the actual process by which the optimal feasible solution is found, the simplex method, may be initiated. The simplex method is an iterative procedure that begins with an initial basic feasible solution; then finds another basic feasible solution that yields a larger value to the objective function. The procedure continues iteratively, moving from one basic feasible solution to another, each time increasing the value of the objective function until a basic feasible solution is reached that provides $z_j - c_j \geq 0$ for all j.

In order to find the optimal feasible solution, the optimality criterion is such that if a basis, B, provides $z_j - c_j \geq 0$ for every j, then B is an optimal basis. No other feasible basis provides a larger value to the objective function [Ladd, 25, p. 6-10].

Let $B_{(t)}$ be the basis in the t-th step. Then

3.22 $$B_{(t)} X_{B(t)} = A_o$$

3.23 $$X_{B(t)} = B^{-1}_{(t)} A_o$$

3.24 $$Z_{B(t)} = c'_{B(t)} B^{-1}_{(t)} A_o$$

$$= c'_{B(t)} X_{B(t)}$$

and

3.25 $$z_{j(t)} - c_j = c'_{B(t)} B^{-1}_{(t)} A_j - c_j$$

$$= c'_{B(t)} X_{j(t)} - c_j$$

from equations 3.16, 3.17, 3.18, and 3.21, respectively. Since the submatrix I defined in equation 3.11 fits the conditions of having m linearly independent vectors that can form a basis in m space, the submatrix, I_m, forms the initial basis from which to start to investigate feasible solutions in finding the optimal feasible solution. This is shown as

3.26 $\quad B_{(t)} = I_m$

where t = 1 or the beginning such that by substitution into equation 3.22

3.27 $\quad B_{(1)}\ X_{B(1)} = A_o$

3.28 $\quad I_m\ X_{B(1)} = A_o$

Certain rules must be used in order to systematically find $B_{(t+1)}$ from $B_{(t)}$ and $B_{(t+2)}$ from $B_{(t+1)}$, etc., until B_o, the optimal feasible basis is found. A rule which works quite well in practice is to use the most negative $z_j - c_j$ to determine A_k (the column vector that enters the basis in order to find $B_{(t+1)}$). This is shown as

3.29 $\quad z_k - c_k = \min_j\ (z_j - c_j),\ z_j - c_j < 0$

Notice, with c_j coefficients of 0 for the slack variables and with c_j coefficients of M (where M is highly negative) on the artificial variables, it is impossible to have the artificial variables enter the optimal feasible solution. Again, the only purpose of the artificial variables is to help in finding the optimal feasible solution, not to be a part of it.

By defining

$$X_{j(t)} = \begin{pmatrix} x_{1j} \\ x_{2j} \\ \vdots \\ x_{mj} \end{pmatrix} \qquad X_{o(t)} = \begin{pmatrix} x_{1o} \\ x_{2o} \\ \vdots \\ x_{mo} \end{pmatrix}$$

there is another rule which works quite well in practice to use in finding A_r (the row vector of $B_{(t)}$ which leaves the basis in order to find $B_{(t+1)}$). This is shown as

3.30 $$\frac{x_{ro}}{x_{rk}} = \min_i \frac{x_{io}}{x_{ik}},\ x_{ik} > 0$$

where x_{ro} is the r-th element of X_o where $X_o = B^{-1}_{(t)}\ A_o$

x_{rk} is the r-th element of X_k where $X_k = B^{-1}_{(t)}\ A_k$

x_{io} is the i-th element of X_o

x_{ik} is the i-th element of X_k

A_r is thus found by finding the minimum x_{io}/x_{ik} which indicates the row vector that becomes A_r.

Once A_r and A_k have been identified, $B^{-1}_{(t+1)}$ must be found. Elements of $B^{-1}_{(t+1)}$ can be obtained from elements of $B^{-1}_{(t)}$ by using equations 3.31 and 3.32 on the elements of $(X_{m_o+1(t)}, X_{m_o+2(t)}, \ldots, X_{m(t)})$, where $(X_{m_o+1(1)}, X_{m_o+2(1)}, \ldots, X_{m(1)}) = I_m$ and t = 1 in the beginning step.

3.31 $$x^{*}_{rj} = x_{rj}/x_{rk} \text{ for } i = r$$

where x_{rj} is the r-th element of $X_j = B^{-1}_{(t)} X_j$

x_{rk} is the r-th element of X_k

3.32 $$x^{*}_{ij} = x_{ij} - \frac{x_{rj}}{x_{rk}} x_{ik} \text{ for } i \neq r$$

where x_{ij} is the i-th element of $X_j = B^{-1}_{(t)} X_j$

x_{rj} and x_{rk} are the same as in equation 3.31

In both equations keep in mind j only represents the columns (m_2+1) through m since these columns represent the initial basis $B_{(1)} = I_m$.

Once the inverse of the new basis, $B^{-1}_{(t+1)}$, has been found using equations 3.31 and 3.32, $z_{j(t+1)}$ can also be determined using equation 3.20. If all $z_{j(t+1)} - c_j \geq 0$, $B_{(t+1)}$ equals B_o, the optimal basis, and the optimal feasible solution can be found

3.23 (a) $$X_o = B^{-1}_o A_o$$

3.24 (a) $$Z_o = c'_{Bo} B^{-1}_o A_o$$

If any $z_{j(t+1)} - c_j < 0$, rules shown in equations 3.29 and 3.30 are used to determine new values of A_r and A_k; i.e., determine the vector, A_r, to leave $B_{(t+1)}$ and the vector, A_k, to replace A_r to obtain $B_{(t+2)}$. Then equations 3.31 and 3.32 are used, the $z_{j(t+2)} - c_j$ are computed, and the process continues.

3. Solution information

Assuming the solution procedure is followed and an optimal basic feasible solution is found, there is much information that can be found in the optimal feasible solution. Three of the most important pieces of information found are the optimal mix of activities (those variables that are multiplied by the m vectors in the final basis), which also can be called the optimal vector, the levels or values the variables hold, and the optimum value of the objective function (also called the value of the program or the objective function value).

Applying these pieces of information to the linear programming theory of the firm, the optimal mix of activities indicate which activities (or processes) to use in the production of outputs. The levels or values the optimal mix of activities hold indicate the number of units of output to produce by each activity (or process). The optimal value of the objective function indicates the maximum level of profit the firm can generate with the available inputs indicated in the problem.

The profit that is indicated by the optimal value of the objective function may have varying interpretations, depending upon the original problem. The profit may be interpreted as income over variable costs, income over variable costs and some fixed costs, or income over total costs, depending upon how the problem is structured. Fixed costs represent constant costs the firm incurs because of the firm's fixed inputs. Thus, any adjustment that must be made because of fixed costs, has no effect on the maximized value other than by the constant value of the adjustment. This can be shown by equation 3.33.

3.33 $\text{Max } (y+k) = (\text{Max } y) + k$

where y is income over variable costs

k is some constant or fixed costs

Fixed costs, however, will not affect the derivation of economic values of traits, provided they are handled in the same manner throughout the derivation procedure.

Another piece of information found in the optimal feasible solution is the amounts of fixed inputs used and not used. With less than or equal to constraints, it is not necessary to use all of the inputs available to obtain an optimal solution. Therefore, applying this to the linear programming theory of the firm, the optimal feasible solution shows the amount of each fixed input used by the activities in producing the different outputs and also the amount of fixed input not used (which is possible because of the introduction of the slack variables).

The final two pieces of information are what are termed shadow prices. The first type of shadow price is called the "income penalty" for an activity. The "income penalty" indicates the amount that income will decrease by requiring the production of one unit of output by an activity not in the optimal mix of activities.

Given by equation 3.21, for $j \leq n$, i.e., real variables

3.21 $z_j - c_j = c'_B B^{-1} A_j - c_j$

and for $j = n + i$

3.21 (a) $$z_{n+i} - c_{n+i} = c'_B B^{-1} A_{n+i} - c_{n+i}$$

The $z_j - c_j$ and $z_{n+i} - c_{n+i}$ are sometimes referred to as criterion elements. If B_o is an optimal feasible basis, then $z_j - c_j \geq 0$ for all j and $z_{n+i} - c_{n+i} \geq 0$ for all i. If X_b (b = 1, 2, ..., n+m) is a basic variable, $x_b \geq 0$ and $z_b - c_b = 0$.

The criterion elements for non-basic real variables are used to find income penalties. All variables not in the basis have a zero value. The criterion elements for non-basic real variables indicate what happens to the value of the objective function if some non-basic real variable is forced into the solution.

In order for a feasible solution to be maintained with the introduction of some non-basic real variable into the solution, the basic variables must change in value. The total change in the objective function for a unit change in a non-basic real variable, x_d, where the maximum value of the objective function is written

3.34 $$Z_o = \sum_{i \in B} c_i x_i$$

where $\sum_{i \in B}$ denotes summation over all variables in the basis B,

can be written as

3.35 $$\frac{dZ_o}{dx_d} = \frac{\partial Z_o}{\partial x_d} + \sum_{i \in B} \frac{\partial Z_o}{\partial x_i} \frac{\partial x_i}{\partial x_d}$$

It can be shown that

$$3.36 \qquad \frac{dZ_o}{dx_d} = -(z_d - c_d) \geq 0$$

[Ladd, 25, pp. 6-33 and 6-34]. Thus, it can be seen that the criterion element for a non-basic real variable shows the change in the objective function that would result from forcing the non-basic real variable into the solution at a value of one and is termed income penalty.

The other type of shadow price is termed the "marginal value product of a fixed resource." The "marginal value product of a fixed resource" indicates the amount that income will decrease if one less unit of input is available for production.

The criterion elements for slack variables are used to find marginal value products of fixed resources. Note in expressions 3.8 (1), (2), and (3), that there is one slack variable appearing in the i-th constraint. This indicates that x_{n+i} corresponds to a_{io} for each i. The criterion elements for slack variables indicate what happens to the value of the objective function if there is some small change in a_{io}. This is shown by

$$3.37 \qquad \frac{\partial Z_o}{\partial a_{io}} = z_{n+i} - c_{n+i} \geq 0$$

[Ladd, 25, pp. 9-5 and 9-11]. Thus, it is now known that the criterion elements for slack variables are termed marginal value products of fixed resources, also called marginal value products of fixed inputs.

Applying these shadow prices to the linear programming theory of the firm, the income penalty indicates the decrease in the profit of the firm provided the firm produces a unit of output by an activity (or process) that is not included in the optimal mix of activities. This decrease in profit results because the firm is sacrificing the production of a different unit of output under a different activity that generates higher returns than the unit of output the firm is determined to produce. The marginal value product of a fixed input indicates the decrease in the profit of the firm which results from a unit decrease in the amount of a fixed input that is available for the production of outputs by the firm.

4. Example solution

Using the same linear programming theory of the firm example problem as described in section III.C., it is easy to show the solution. In order to refresh the memory, the problem was set up as

$$\text{Max } 123.35\ x_1 + 112.66\ x_2 + 105.80\ x_3$$

subject to

$$40\ x_1 + 50\ x_2 + 35\ x_3 \leq 11{,}000 \text{ (corn)}$$
$$3.25\ x_1 + 0.72\ x_2 + 2.0\ x_3 \leq 900 \text{ (silage)}$$
$$0.11\ x_1 + 0.25\ x_2 + 0.3\ x_3 \leq 300 \text{ (hay)}$$
$$6.0\ x_1 + 6.4\ x_2 + 5.0\ x_3 \leq 1600 \text{ (labor)}$$
$$x_1,\ x_2,\ x_3 \geq 0$$

where x_1 represented activity #1 or buying feeder calves, feeding them a high roughage ration, and then selling them for slaughter, x_2

represented activity #2 or buying feeder calves, feeding them a high grain ration, and then selling them for slaughter, and x_3 represented activity #3 or buying yearling steers, feeding them a medium roughage-medium grain ration, and then selling them for slaughter.

After solving the problem, using the simplex method, the following solution was found:

Optimal mix of activities (or optimal vector)

x_1; produce 100 steers under activity #1

x_2; produce 0 steers under activity #2

x_3; produce 200 steers under activity #3

Income over variable costs = \$33,511

Input use:

Fixed inputs used

11,000 bushels of corn

725 tons of silage

71 tons of hay

1,600 hours of labor

Fixed inputs unused

175 tons of silage

229 tons of hay

Variable inputs purchased and used

The variable inputs purchased at a constant price and used in the production of outputs may indirectly be found from the solution by multiplying the optimal level of production of each activity (i.e., the levels of x_1, x_2, and x_3) times the production coefficient of variable input, q_{kj}, found in equation 3.1 for each respective activity and then summing over j. This is shown as

$$\text{k-th variable input use} = \sum_{j=1}^{n} x_j \, q_{kj}$$

Also remember that the firm only purchases the amounts of variable inputs it will use in the production of outputs.

Shadow prices:

Income penalties of an activity

If one steer was produced under activity #2, there would be a decrease in profit of $32.50.

Marginal value product of a fixed input

If one less bushel of corn were available for the production of outputs, there would be a decrease in profit of $1.85.

If one less hour of labor were available for the production of outputs, there would be a decrease in profit of $8.21.

E. Sensitivity Analysis

Investigations that deal with changes in an optimal feasible solution due to changes in the parameters of the linear program are termed sensitivity analyses. In this section of the chapter, sensitivity analysis will be used to examine the sensitivity of the optimal value of the objective function to changes in the parameters of the linear program.

1. Application in animal breeding

In section I.A., pp. 3-4, it was stated that "the individual's phenotype is a result of the influence of many genes and other factors" and was shown mathematically as

$$1.1 \qquad P = G + E$$

where P = phenotypic value of an individual; measurable
G = genotypic value of an individual; non-measurable
E = environmental deviation

It was also stated that "by investigating characters or traits of an individual such as hair color, eye color, or feed efficiency in livestock, much can be learned about gene influences on individuals and the genetic make-up of populations" (section I.A., p. 4). Looking at equation 1.1 more closely, it can be seen that if environment remains constant, changes in genotype are exactly shown in the phenotype of the individual and thereby seen in the individual's traits or characters.

In constructing a linear program for the firm, certain assumptions must be made with respect to environment and phenotype (the overall traits or characters) of the livestock that the farm possesses. From this it follows that a certain genotype is assumed for the livestock of the farm. The assumptions must be made so as to develop the input-output coefficients for the two types of inputs and also the prices received for the livestock when sold; all parameters of the linear program. The production coefficients of the inputs and the prices received for the livestock vary, depending on the quality of the livestock (or the genotype of the livestock).

Economic values used in the selection index (or the economic values of traits) are defined as the amount by which profit of the firm may be expected to increase for each unit of improvement in a trait of a single animal. It follows that by finding the profit of the firm given the environmental and phenotypic assumptions and then finding a new profit of the firm given the same environmental assumptions, but different phenotypic assumptions concerning one trait, the difference in profit should reflect the economic value of the trait. The acceptance of the sentence above is the basis for the method of finding economic values of traits that is to be presented in this thesis.

Sensitivity analysis works well with the idea of changing phenotypic assumptions. By changing parameters in the linear program, thereby reflecting changes in the phenotypic assumptions, the sensitivity of the objective function or in the case of the firm, the change in profit, is shown, giving the economic value of the trait that was assumed to be changed. By changing certain parameters in the linear program so as to reflect a change of a certain trait (or genotype, since environmental conditions remain constant), the change in profit reflects the economic value of the certain trait.

Since different parameters may be changed because of changing different traits, different cases may arise in sensitivity analyses. Three general cases may be defined in sensitivity analysis in finding economic value of traits. These are defined as Case I, Case II, and

Case III, and are given in the following. Each case is the result of having to change parameters of the linear program due to the improvement of a trait by one unit.

a. Case I Case I involves changing the parameter c_j or the coefficient of the objective function which defines the net return of the j-th activity to the firm. Thinking back, it has been said that c_j is found by defining the price received for the output by the j-th activity and subtracting from it the variable costs associated with the production of output by the j-th activity, which are found by multiplying the price of each variable input times the production coefficient of the variable input and summing over all variable inputs or

$$p_j - \sum_{k=1}^{n} r_k \, q_{kj} = c_j$$

as defined before in equation 3.2.

Immediately it can be seen that Case I can be broken down into subcases since c_j may be changed by changing p_j, the price received when selling the j-th output, q_{kj}, the production coefficient of the variable input, or a simultaneous change in p_j and q_{kj}. These will be respectively defined as Case IA, Case IB, and Case IC.

(1) Case IA In this case, the change in c_j, net return to the firm for the j-th activity, results from changing the price received for the output of the j-th activity as a result of the improvement of a trait by one unit. Looking at equation 3.2, it can be seen that by increasing the price received for the output of the j-th activity, c_j is increased. An example of such a case

would be a higher price received for a market hog because of less backfat.

(2) Case IB In this case, the change in c_j results from changing the production coefficient of variable inputs in the j-th activity due to reductions in the amounts of variable inputs used per unit of output of the j-th activity as a result of the improvement of a trait by one unit. Looking at equation 3.2, it can be seen that by decreasing the production coefficient of variable input, q_{kj}, the total cost of the k-th variable input used in the production of output by the j-th activity is decreased, thereby decreasing the total variable costs, $\sum_k r_k q_{kj}$, and thereby increasing the net revenue of the j-th output produced by the j-th activity, c_j. An example of such a case would be less electrical cost due to better mothering ability (where the baby pigs would be stronger and may not need heat lamps for as long a time).

(3) Case IC In this case, the change in c_j, net return to the firm for the j-th activity, results from simultaneously changing the price received for the output of the j-th activity and the production coefficient of variable inputs used for the production of output by the j-th activity. In other words, this case demonstrates the possibility of Case IA and Case IB occurring simultaneously. An example of such a case would be a higher price for a breeding animal because of improved feet and legs and less variable costs (due to less variable inputs needed for production) due to less care of the animal because of the improved feet and legs.

There is no case where there is a change in profit due to a change in the price of the k-th variable input, r_k, because it is assumed that the variable inputs are available in unlimited supply at a constant price.

b. Case II Case II involves changing a parameter a_{ij}, or the production coefficient of fixed inputs which defines the amount of the i-th fixed input needed to produce one unit of output by the j-th activity. This case is similar to Case IB in that there is a change in production coefficient due to the reduction in the amount of input needed per unit of output produced in the j-th activity. The difference comes in that in this case, there is a change in the production coefficient of fixed inputs, thereby changing the amount of fixed inputs used per unit of output of the j-th activity.

An example can be shown if it is assumed that a trait is improved by one unit and it is also assumed that the trait is feed efficiency. Also suppose the j-th activity is feeding and selling market hogs. Now, when the feed efficiency trait is improved by one unit, the amount of feed used, assuming all feed ingredients are fixed inputs (which may not always be the case), is reduced while the net revenue received for marketing the hogs remains the same. This results in a decrease in some of the a_{ij}'s and a zero change in the c_j's. So, in this case example of feeding and selling market hogs, the amounts of corn, soybean oilmeal, etc., fed to the hogs are reduced while the net revenue received for marketing each hog remains the same.

c. Case III Case III involves changing the parameters c_j and a_{ij}, simultaneously. In other words, Case III demonstrates the possibility of Case I and Case II occurring simultaneously. Due to the breakdown of possibilities of changing the c_j parameter, Case III, as Case I, is broken down into subcases. These will be respectively defined as Case IIIA, Case IIIB, and Case IIIC.

(1) Case IIIA This is a case where Case IA and Case II occur simultaneously. The change in the net return of the output of the j-th activity results from changing the price received by selling a unit of output by the j-th activity. The change in the production coefficient of fixed input results from changing the amount of a fixed input used per unit of output by the j-th activity. The changes in the net return and production coefficients of fixed inputs occur simultaneously and come as a result of a unit change of improvement in a trait. An example of such a case would be the selling of breeding stock where the animal is sold at a premium because of its superior genetic make-up while consuming less fixed inputs (e.g., feed) in its production while on the farm.

(2) Case IIIB This is a case where Case IB and Case II occur simultaneously. The change in the net return of the output of the j-th activity results from changing the production coefficients of variable inputs due to the change in the amount of the k-th variable input used per unit of output produced in the j-th activity and the change in the production coefficient of fixed inputs results from changing the amount of a fixed input used per unit of output of the j-th activity. The changes in the amounts of fixed and variable

inputs used occur simultaneously and come as a result of a unit change of improvement in a trait. An example of such a case would be higher average daily gain where the animal would use less fixed inputs (e.g., labor) and less variable inputs (e.g., electricity) due to a shorter feeding period before selling.

(3) Case IIIC This is a case where Case IC and Case II occur simultaneously. The change in the net return of the output produced in the j-th activity results from changing both the price received by selling a unit of the j-th output and the production coefficient of variable inputs used in producing the j-th output. The change in the production coefficient of fixed inputs results from changing the amount of a fixed input used per unit of output produced by the j-th activity. The changes causing a change in the net revenue of the j-th output and the change causing the change in the production coefficients of fixed inputs occur simultaneously and come as a result of a unit change of improvement in a trait. An example of such a case would be improved weaning weight in calves where the calves, if sold, would have a higher selling price, a lower amount of variable inputs used (e.g., lower amount of supplement in creep feed assuming supplement here is a variable input), and a lower amount of fixed inputs used (e.g., lower amount of corn in creep feed assuming corn is a fixed input).

2. Symbolic representation of sensitivity analysis

It is possible, using symbolic representation, to demonstrate the use of sensitivity analysis in finding economic values of traits. Using implicit functions, differential calculus, and previously defined concepts, the idea of finding economic values of traits can be easily presented.

It has been previously shown in equation 3.5 that $\sum_{j=1}^{n} c_j x_j$ is the objective function that is optimized. Now, assume that by using the simplex method, an optimal objective function is found and shown as

$$3.38 \qquad \sum_j c_j \, x_{jo}$$

where x_{jo} is the value held by the variable j in the optimal mix of activities. Also, assume that

$$3.39 \qquad \sum_j c_j \, x_{jo} = Z$$

From previous explanations of linear programming, it is known that values of the x_{jo} are the optimal levels of each of the J activities found as a function of the production coefficients of fixed inputs (a_{ij}), the levels of available fixed inputs (a_{io}), and the net returns of each of the J activities (c_j). This can be shown as

$$3.40 \qquad x_{jo} = g_j \, (a_{11}, a_{12}, \ldots, a_{21}, a_{22}, a_{23}, \ldots, a_{mn}, a_{1o}, a_{2o}, \ldots, a_{mo}, c_1, c_2, \ldots, c_n)$$

where x_{jo} is a function g_j of the parameters: the production coefficients of fixed inputs, the available fixed input levels, and the net returns of each activity.

By substituting equation 3.40 into equation 3.39, it can be seen
:

3.41 $$Z = \sum_{j=1}^{n} c_j \left[g_j \left(a_{11}, a_{12}, \ldots, a_{21}, a_{22}, a_{23}, \ldots, a_{mn}, a_{1o}, a_{2o}, \ldots, a_{mo}, c_1, c_2, \ldots, c_n \right) \right]$$

examining equation 3.41, it is possible to see that Z, the value the objective function, is a function of the c_j, the net returns the J activities; the a_{ij}, the production coefficients of fixed ıputs; and the a_{io}, the levels of fixed inputs.

It has been alluded to earlier that changes in the traits of the ivestock owned by the farm cause certain parameters of the linear rogram of the farm to vary. From this it can be said that certain arameters of the linear program of the farm are functions of traits. hese parameters are shown in the following equations as functions f the h-th trait,

3.42 $$a_{ij} = \emptyset_{ij}(t_h)$$

3.43 $$p_j = \Psi_j(t_h)$$

3.44 $$q_{kj} = \rho_{kj}(t_h)$$

where a_{ij} is a function, $\emptyset_{ij}$, of the h-th trait (t_h)

p_j is a function, Ψ_j, of the h-th trait (t_h)

q_{kj} is a function, ρ_{kj}, of the h-th trait (t_h)

although every a_{ij} may not be a function, ϕ_{ij}, of the h-th trait (t_h), every p_j may not be a function, Ψ_j, of the h-th trait (t_h), and every q_{kj} may not be a function, ρ_{kj}, of the h-th trait (t_h). It is only with knowledge of animal breeding and livestock production that distinctions can be made between those a_{ij}, p_j, and q_{kj} parameters that are functions of the h-th trait (t_h), and those a_{ij}, p_j, and q_{kj} parameters that are not functions of the h-th trait (t_h).

p_j and q_{kj} are shown as functions of the h-th trait so as to clarify the relationship that c_j is in fact a function of the h-th trait, since

$$c_j = p_j - \sum_{k=1}^{u} r_k \, q_{kj}$$

3.45 $$c_j = \Psi(t_h) - \sum_{k=1}^{u} r_k \, \rho_{kj}(t_h)$$

The parameters a_{io} and r_k are not affected by the h-th trait, t_h, since a_{io} is the level of fixed inputs and is stable by assumption and since r_k is the <u>constant</u> price paid in purchasing a unit of the k-th variable input.

Since the economic value of traits has been defined as the amount by which the profit of the firm may be expected to increase for each unit of improvement in the trait of a single animal, by using previously defined relationships and differential calculus, the change in the profit of the firm due to a change in the h-th trait may be shown in the following equations.

Given equation 3.41, the change in profit of the firm due to a change in the h-th trait, when the various c_j are the only parameters affected by the change in the h-th trait, is

$$3.46 \qquad dZ/dt_h = \sum_{j=1}^{n} \partial Z/\partial c_j \; dc_j/dt_h \qquad \text{(Case I)}$$

where dt_h is the change in the h-th trait, dc_j is the change in net return per unit of the j-th output, $\partial Z/\partial c_j$ is the (partial) change in the profit of the firm due to the change in the net return of one unit of the j-th output, and dZ/dt_h is the total change in the profit of the firm due to the change in the h-th trait (summed over all n activities).

As shown before in equation 3.45, since

$$p_j = \Psi_j(t_h)$$

$$q_{kj} = \rho_{kj}(t_h)$$

c_j may be affected in three different ways: by changing only p_j, by changing only q_{kj}, or by changing p_j and q_{kj} simultaneously. These were explained in Case IA, Case IB, and Case IC, and can be shown by equation 3.46. By defining dc_j/dt_h of equation 3.46 differently, the three cases may be seen.

When

$$dc_j/dt_h = dp_j/dt_h - \sum_{k=1}^{u} r_k \, q_{kj}$$

where dp_j/dt_h is the change in the price received when selling a unit of output produced by the j-th activity and results from the improvement of the h-th trait, Case IA can be demonstrated.

When

$$dc_j/dt_h = p_j - \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$$

where dq_{kj}/dt_h is the change in the production coefficient of variable inputs that is a result of the improvement of the h-th trait, Case IB can be demonstrated.

When

$$dc_j/dt_h = dp_j/dt_h - \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$$

where dp_j/dt_h and dq_{kj}/dt_h are defined as before, Case IC can be demonstrated.

Again, given equation 3.41, the change in profit of the firm due to a change in the h-th trait, when the a_{ij} are the <u>only</u> parameters affected by the change in the h-th trait, is

$$3.47 \qquad dZ/dt_h = \sum_{i,j=1}^{m,n} \partial Z/\partial a_{ij} \, da_{ij}/dt_h \qquad \text{(Case II)}$$

where dt_h is the change in the h-th trait, da_{ij} is the change in the production coefficient of fixed input i for the j-th activity, $\partial Z/\partial a_{ij}$ is the (partial) change in the profit of the firm due to the partial change in the production coefficient of fixed inputs, and dZ/dt_h is the total change in the profit of the firm due to the change in the h-th trait (summed over all m fixed inputs and n activities).

Again, given equation 3.41 where the change in profit of the firm due to a change in the h-th trait, when the c_j and a_{ij} are simultaneously affected by the change in the h-th trait, is

$$3.48 \qquad dZ/dt_h = \sum_{j=1}^{n} \partial Z/\partial c_j \; dc_j/dt_h + \sum_{k,j=1}^{m,n} \partial Z/\partial a_{ij} \; da_{ij}/dt_h \qquad \text{(Case III)}$$

where the variables are the same as previously defined.

Since Case III, as in Case I, can be broken down into subcases by defining dc_j/dt_h differently, the following is true.

When

$$dc_j/dt_h = dp_j/dt_h - \sum_{k=1}^{u} r_k \, q_{kj}$$

as in Case IA, then by plugging $dp_j/dt_h - \sum_{k=1}^{u} r_k \, q_{kj}$ into equation 3.48, Case IIIA can be demonstrated.

When

$$dc_j/dt_h = p_j - \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$$

as in Case IB, then by plugging $p_j - \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$ into equation 3.48, Case IIIB can be demonstrated.

When

$$dc_j/dt_h = dp_j/dt_h - \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$$

as in Case IC, then by plugging $dp_j/dt_h - \sum_{k=1}^{u} dq_{kj}/dt_h$ into equation 3.48, Case IIIC can be demonstrated.

When Z is considered as a function of c_j, a_{ij}, and a_{io}, and when a_{io} is considered constant, and also when c_j and a_{ij} are considered functions of an implicit parameter, t_h, the total derivative of Z with respect to t_h is given by

$$3.49 \qquad dZ/dt_h = \sum_{j=1}^{n} \partial Z/\partial c_j \, dc_j/dt_h + \sum_{i,j=1}^{m,n} \partial Z/\partial a_{ij} \, da_{ij}/dt_h$$

where $dc_j/dt_h = dp_j/dt_h + \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$ and r_k is considered constant.

Continuing on with the assumption that a_{io} and r_k are constant and a_{ij} and q_{kj} and p_j (thus c_j) are variable, equation 3.49, through manipulation, may be converted into a computable form [Gass, 10, p. 152] as follows:

$$3.50 \qquad dZ/dt_h = -\sum_{i,j=1}^{m,n} \partial Z/\partial a_{io} \, \partial Z/\partial c_j \, da_{ij}/dt_h + \sum_{j=1}^{n} \partial Z/\partial c_j \, dc_j/dt_h$$

where $dc_j/dt_h = dp_j/dt_h + \sum_{k=1}^{u} r_k \, dq_{kj}/dt_h$ and where $\partial Z/\partial a_{io}$ is defined as equal to $z_{n+i} - c_{n+i}$ or the value marginal product of fixed input i as defined in equation 3.37 and $\partial Z/\partial c_j = x_{jo}$, the optimal solution values for basic variables, by partially differentiating equation 3.33 with respect to c_j.

Equation 3.50 provides the sensitivity of the objective function (the change in profit of the firm caused by a unit change in the h-th trait, dZ/dt_h) with regard to an implicit parameter, t_h. This is true, though, only if the optimal mix of activities remains the same, even with the parameter changes. As long as dt_h, the change in the h-th trait, is small enough so as not to cause a change in the optimal mix of activities in solution, equation 3.50 will provide an accurate change in the profit of the firm. If the change in the h-th trait is too large, the optimal mix of activities will change and dZ/dt_h, the change in the profit of the firm due to a change in the h-th trait from the computable form equation 3.50, will become inaccurate.

When the change in the h-th trait is too large in that it changes the optimal mix of activities and thus causes inaccuracy in the change in profit found using the computable form, other alternatives for finding the change in profit are available. One alternative available is to solve another maximization linear program. By changing the parameters in the first linear program, to reflect a change in the trait, and leaving the remaining parameters unchanged, a second linear program can be solved. By simply finding the difference of the profit of the first linear program and the profit of the second linear program, the change in profit due to a change in the trait can be found. (This method works whether the optimal mix of activities changes or not.)

F. The Computable Form

Writing the computable form of the change in profit for a unit change in the h-th trait, again, it is known that

$$dZ/dt_h = -\sum_{i,j=1}^{m,n} \partial Z/\partial a_{io}\ \partial Z/\partial c_j\ da_{ij}/dt_h + \sum_{j=1}^{n} \partial Z/\partial c_j\ dc_j/dt_h$$

where $\partial Z/\partial a_{io}$ is the marginal value product of fixed input i (or the criterion elements of slack variables of the primal)

$\partial Z/\partial c_j$ is the optimal solution values for basic variables, X_{jo}

da_{ij}/dt_h is the change in the production coefficient of the i-th fixed input due to the unit change in the h-th trait

dc_j/dt_h is the change in the net return per unit of the j-th output due to the unit change in the h-th trait

dZ/dt_h is the change in profit of the firm for a unit change in the h-th trait

Thus, it can alternatively be shown as

3.50 (a) $$dZ/dt_h = -\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\ x_{jo}\ da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\ dc_j/dt_h$$

Writing the definition of economic values of traits, again, it is written: The economic value of a trait is the amount by which profit of the firm may be expected to increase for each unit of improvement in a trait of a single animal.

In comparing the definition of economic value of a trait and the economic value that the computable form of profit differentiation gives, it can be seen that they are not identical. The economic value of a trait must be "of a single animal," the economic value that the computable form gives is not "of a single animal," but aggregated over all animals (output) that are affected by the unit change of the h-th trait (or the parameter changes of the linear program). In other words, the computable form does not give the change in profit of the firm expected with a unit change of improvement in a trait of a single animal, but gives the change in the profit of the firm expected with a unit change of improvement in the same trait of every animal produced by the farm with the improved trait.

The computable form may be altered, though, so that it will reveal the true economic value of a trait. By dividing dZ/dt_h by Σx_{j*} where j* identifies every activity that produces an animal that has a unit improvement in the h-th trait, the true economic value of the h-th trait will be found. This revised computable form is shown as

3.51 $$E.V. = \left[\frac{1}{\sum_{j*} x_{j*}}\right] \left[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \right]$$

where $\sum_{j*} x_{j*}$ is the number of animals produced by the farm with the h-th trait improved and where j* identifies every activity that produces an animal that has a unit improvement in the h-th trait

E.V. is the economic value of the h-th trait

If levels of livestock activities are measured in numbers of animals, $x_{j*} = x_{jo}$ and $\Sigma x_{j*} = \Sigma x_{jo}$ for those livestock activities that have livestock with a unit improvement in the h-th trait. If levels of livestock activities are measured in hundredweight, x_{j*} equals x_{jo} times the reciprocal of the average weight per head of livestock in hundredweight for those livestock activities that have livestock with a unit improvement in the h-th trait.

G. Summary

Many pieces of information have been presented in this chapter. A linear programming theory of a profit maximizing firm was presented. The solution procedure for solving a linear program was presented. Sensitivity analysis was presented so as to see how a change in profit may be found using the economic model. A computable form of sensitivity analysis was presented to symbolically present sensitivity analysis. Finally, a revised computable form was shown for finding the true economic value of a trait.

To further give the reader perspective, the following can be said about the procedure of finding the economic value of a trait. First, a firm must be developed. After the development of the firm, the firm can be put into a linear program problem by forming basic parameters of the linear program to reflect the firm and thereby developing an economic model of the firm. Using the simplex method, the optimal combination of inputs and output can be found so as to maximize the firm's profit. Then, by substituting into the revised computable form, information from the optimal solution, and by changing certain parameters so as to reflect a change in a trait, the revised computable form will give the economic value of the trait that was to be found.

Although the procedure just described presents a very simple approach to finding the economic value of a trait, it is important to keep in mind the theory and assumptions underlying the procedure. None of the problems within the system have been viewed yet. None of the advantages of the system have been viewed yet. An actual working economic model has not been viewed yet, either. These things and more will be looked at in the following chapters so that a clearer understanding may be developed for the construction of and demonstration of an economic model which can be used to derive economic values for use in selection indexes.

IV. FINDING ECONOMIC VALUES: AN EMPIRICAL ANALYSIS

In the previous sections, an economic model which can be used to derive economic values of traits was derived by looking at the maximization linear program and then sensitivity analysis and the revised computable form. A clearer understanding of the economic model may be found by an empirical analysis.

In this section, an empirical economic model of a swine farm will be used to derive economic values of certain swine traits. In order to provide a full understanding of deriving economic values, this chapter will look at the linear program of a swine farm, view the optimal solution of the linear program, and use the revised computable form to determine economic values.

A. Model I

1. General description of the swine farm

Prior to developing any linear program of a farm firm, it is necessary to decide upon the processes that take place within the farm. The farm to be developed in this section will be a specialized farm; its only marketed products are swine. The type of activities of the swine farm must therefore be decided upon.

The swine farm to be developed in this section will be flexible. It will have four farrowing activities and four feeder pig buying activities so that it may farrow, may buy feeder pigs, or both. Farrowing times are in May, August, November, and February. Feeder pigs may be purchased in June, September, December, and March.

If the farm farrows pigs, the females that farrow may come from various sources. The farm will have the option of purchasing new gilts for each farrowing or raising gilts for each farrowing. The farm, though, will allow gilts that farrow in May and August to farrow again in November and February, respectively. Gilts that do not conceive or are culled prior to the second farrowing are marketed and replaced by newly purchased or raised gilts. Females that farrow in November and February are marketed following the weaning of their pigs. Gilts that do not conceive for farrowing in May and August are also marketed.

Two boars will be purchased in November to breed the gilts and sows that farrow in May, August, November, and February. In the following November, the boars will be marketed since they will have served their purpose by then.

Pigs that are farrowed and feeder pigs that are purchased will be fed to weights of 180, 200, 220, 240, or 260 pounds. The weight to which the pigs will be fed will be dependent upon profitability of the different weights. Since there are four farrowings and four possible times to purchase feeder pigs, and also five possible market weights, there will be 20 possible times to market finished hogs.

Other activities will be developed, such as preparing newly purchased gilts and raised gilts for farrowing, feeding the boars, feeding weaned pigs to 40 pounds, and feeding 40 pound pigs to optimal marketing weights. The development of the activities within the linear program, though, is partially dependent upon basic assumptions that must be made about the swine farm. These assumptions will be looked at in closer detail in the following part of the model section.

2. Assumptions

In addition to the basic assumptions of linear programming explained in section III.A.2, certain additional assumptions must be made in the development of any empirical linear program of a farm firm. These additional assumptions aid in developing the coefficients of the linear program and allow for a systematic procedure in finding economic values of traits.

a. Technology (Assumption 1) Technology must be known so that the methods of production of outputs are known. The level of technology must remain constant, though. Any change in the level of technology assumed while developing the linear program may distort the economic values of the traits.

The farm firm has a central farrowing house that is fully insulated and winter environmentally controlled. It has a 25 sow capacity and is equipped with farrowing stalls, feed and water troughs in the stalls for both sow and baby pigs, and manure handling facilities.

The farm also has partial confinement growing-finishing units that consist of two open-front buildings, concrete floors extending in front of both buildings to provide areas for exercise and feeding, self-feeders (one for every 50 head of pigs), and heated, automatic waterers.

Each partial confinement building has 3,250 square feet of housing area. This area is sufficient to house 250 head of 220 pound market hogs during the summer. This also allows the total number of pigs weaned, from sows farrowed in the central farrowing house, to be housed within the partial confinement facilities.

The buildings are completely enclosed except for the front which has partially closeable doors. The buildings are not insulated but have composition roofs for condensation control. The buildings and feeding floors are divided into narrow pens with little bedding used except in cold weather. Manure is allowed to accumulate at the lower end of the feeding floor before being hauled to the field.

b. Environmental conditions (Assumption 2) Environmental conditions must be known so that the quantities of inputs needed to produce a certain quantity of output are known. The amounts of inputs needed to finish market animals are dependent upon the severity of the environmental conditions.

The environmental conditions that are assumed in developing the linear program of the swine enterprise can be termed moderate. The four seasons of the year are assumed to be evident, each season being temperate. Other environmental conditions due to management, geographic location, etc., are assumed to be typical for a Midwest swine farm.

c. Period length (Assumption 3) The length of time with which the linear program is involved is assumed so as to aid in determining such things as the number of times certain activities take place, cyclical or seasonal price variations and when and how they affect the sale of output, whether it is necessary to discount prices or net revenues to present value, or even whether a multi-period linear program may be more appropriate [Loftsgard and Heady, 30, p. 51].

The length of time that is assumed here is a 22 month period: from November 1, 1972, through August 31, 1974. The 22 month period represents the time period in which sequential activities associated with a swine farm (i.e., purchasing gilts that farrow through marketing slaughter hogs) farrowing four litters could occur. This, though, will become clearer as the linear program is discussed in more detail.

d. Discounting to present value (Assumption 4) It was alluded to earlier that it may be necessary to discount prices or net revenues to present value, depending on the period length of the linear program. Once the period length of the linear program is decided upon, a decision on whether or not to discount must be reached.

Points to consider in making a decision in regard to discounting are:

(1) Rate of pure time preference Money is assumed to be worth more at the present than in the future. The percentage rate at which money is worth more at the present than in the future is at least one portion of the discount rate.

(2) Rate of inflation Inflation causes money to be worth more at the present than in the future. Deflation causes money to be worth more in the future than at the present. With inflation, the percentage rate at which it occurs may be another portion of the discount rate.

(3) Required rate of return due to risk Return on investments vary greatly with the type of investment. Investments with

high risk usually require higher rates of return than investments with low risk. The difference in rates of return on investments is usually due to the degree of risk.

(4) Opportunity cost Certain investments are essentially riskless. These investments have rates of return to account for the rate of pure time preference and the rate of inflation factors. If a person chose to receive a dollar a year from today as opposed to receiving the dollar today, there is an opportunity cost involved, since the person passed up earnings that could be obtained by investing the dollar in essentially riskless investments. The rate of return that the dollar could have earned can also be termed an opportunity cost.

Suppose a farmer invested some money into his swine farm for expansion. He is sure of receiving a 7 percent return on his investment. Also suppose the farmer could invest the money elsewhere, with essentially no risk, and have an assured return of 12 percent. Although the total 12 percent is not an opportunity cost, the farmer has an opportunity cost of 5 percent by not investing the money in the essentially riskless investment. Thus, in order that no opportunity cost prevail, any investment that is made must yield at least the same rate of return as the rate of return on the essentially riskless investments.

Because the rate of pure time preference and the inflation rate within the 22 month period from November 1, 1972, to August 31, 1974, was certainly noticeable, it is almost imperative that a discounting procedure be used. By discounting net revenues of activities, derived economic values will not be biased upward due to inflation and the absence of the rate of pure time preference. The exact discounting procedure used, though, will be discussed later in detail.

The linear programming theory of the firm incorporates an assumption that the firm already has a certain set of fixed inputs available to use in the production of output. From this standpoint, the firm has made at least a partial commitment in regard to investment and thereby partially eliminates the consideration of the risk of investment. By partially disregarding risk, an excellent discount rate to use is opportunity cost. Again, opportunity cost, in this sense, is the assured rate of return on essentially riskless investments accounting for the rate of pure time preference and the rate of inflation.

The opportunity cost or discount rate used in the discounting procedure is 12 percent per annum or 1 percent per month. The 12 percent rate of discount is assumed to be the average rate of return on essentially riskless investments covering any rate of pure time preference and rate of inflation during the 22 month period.

e. Current stage of genetic progress of livestock of the farm firm (Assumption 5) Another assumption that must be made in developing an empirical linear program of a farm firm is the phenotypic measure of the trait for which the economic value is to be derived. The phenotypic measures of the traits must be known so that the relationship of quantities of inputs needed to produce a certain quantity of output are known and/or so that the price received per unit of output can be determined.

The process of deriving economic values will be demonstrated with three traits in this thesis. The traits are backfat, feed efficiency, and average daily gain. Thus, phenotypic measures of each trait must be assumed in order to develop the linear program. These can be seen in table 4.1.

Table 4.1. Assumed phenotypic measures of market hogs[a]

Trait	Weights (pounds)				
	180	200	220	240	260
Backfat (inches)	1.3	1.38	1.46	1.54	1.62
Feed efficiency (feed/pound gain)	3.4143	3.4656	3.5222	3.5850	3.6545
Average daily gain (gain/day)	1.5246	1.5804	1.6298	1.6728	1.7109

[a]Source: Life Cycle Swine Nutrition, 29, p. 7.

f. Fixed inputs available (Assumption 6) In section III.A.1, it was indicated that the "firm has a set of fixed inputs available for use." It is therefore necessary to specify the types and amounts of fixed inputs available. The types and amounts of fixed inputs available for use by the farm firm are shown in tables 4.2a through 4.2d.

g. Rations (Assumption 7) Basic rations are assumed so as to find the amounts of inputs needed to feed the livestock. The four basic rations assumed to be used by the swine farm are shown in tables 4.3a through 4.3d.

To the basic feed ingredients in rations are often added pharmaceutical feed additives. These are added for the prevention

Table 4.2a. Monthly fixed labor inputs during the 22 month period: Model I

Row number	Month	Available hours	Row number	Month	Available hours
1	November 1972	160	12	October 1973	160
2	December 1972	196	13	November 1973	160
3	January 1973	216	14	December 1973	196
4	February 1973	192	15	January 1974	216
5	March 1973	198	16	February 1974	192
6	April 1973	160	17	March 1974	198
7	May 1973	160	18	April 1974	160
8	June 1973	160	19	May 1974	160
9	July 1973	216	20	June 1974	160
10	August 1973	208	21	July 1974	216
11	September 1973	168	22	August 1974	208

Table 4.2b. Farrowing capacity for each farrowing: Model I

Farrowing number	Farrowing month	Number of sows
1	May 1973	25
2	August 1973	25
3	November 1973	25
4	February 1974	25

Table 4.2c. Finishing capacity for market hogs: Model I

Confinement building	Market hog group	Number of square feet available
1	May 1973	3250
2	August 1973	3250
1	November 1973	3250
2	February 1974	3250

Table 4.2d. Number of boars available: Model I

	Number of boars
Boars	2

(and often treatment) of diseases. The pharmaceutical feed additives, their amounts, and the time in which they are added are shown in table 4.4.

h. Prices (Assumption 8) In developing an empirical linear program of a farm firm, it is necessary to make price assumptions for variable inputs and the farm firm's output. As described in section III.A.1, a variable input is an input that is necessary for the production of output as are fixed inputs, but is available in unlimited quantity at a constant price. The farm's output is also sold at a constant price, but the quantity sold is limited by the amount produced by the farm. The price assumptions of variable inputs and the farm firm's output are shown in tables 4.5a and 4.5b.

The prices of feed inputs shown in table 4.5a are average prices during the 22 month period from November 1, 1972, through August 31, 1974. The prices of swine shown in table 4.5a and of outputs shown in table 4.5b are actual prices that occurred given the day that the swine were purchased or sold during the 22 month period.

3. Formation of certain linear program coefficients

Parameters of linear programs were defined in section III.C. These parameters were the level of fixed inputs a_{io}, production coefficients of fixed inputs a_{ij} (input-output coefficients), and

Table 4.3a. Basic ration for pregestation, breeding, and gestation[a]

Input	Units	Intake per day 3 pounds	Intake per day 4 pounds	Intake per day 5+ pounds
Corn	bushels	29.054	31.429	32.732
	pounds	1627	1760	1833
Soybean oilmeal	pounds	250	150	100
Dicalcium phosphate	pounds	70	45	30
Limestone	pounds	15	15	15
Salt	pounds	15	12.5	10
Trace mineral premix	pounds	3	2.5	2
Vitamin premix	pounds	20	15	10
Total	pounds	2000	2000	2000

[a]Source: Feeding and Managing the Swine Breeding Herd, 6, p. 6.

Table 4.3b. Basic ration for lactation[a]

Input	Units	Intake per day Full
Corn	bushels	29.94
	pounds	1677
Soybean oilmeal	pounds	250
Dicalcium phosphate	pounds	25
Limestone	pounds	15
Salt	pounds	10
Trace mineral premix	pounds	3
Vitamin premix	pounds	20
Total	pounds	2000

[a]Source: Life Cycle Swine Nutrition, 29, p. 14.

Table 4.3c. Basic starter ration (18.62 percent protein)[a]

Input	Units	Intake per day Full
Corn	bushels	19.41
	pounds	1087
Soybean oilmeal	pounds	550
Dried whey	pounds	300
Limestone	pounds	10
Dicalcium phosphate	pounds	25
Salt	pounds	5
Trace mineral premix	pounds	3
Vitamin premix	pounds	20
Total	pounds	2000

[a]Source: Life Cycle Swine Nutrition, 29, p. 15.

Table 4.3d. Basic finishing ration (14 percent protein)[a]

Input	Units	Intake per day Full
Corn	bushels	29.18
	pounds	1634.19
Soybean oilmeal	pounds	305.81
Limestone	pounds	15
Dicalcium phosphate	pounds	23
Salt	pounds	10
Trace mineral premix	pounds	2
Vitamin premix	pounds	10
Total	pounds	2000

[a]Source: Life Cycle Swine Nutrition, 29, p. 17.

Table 4.4. Pharmaceutical feed additives and basic rations to which they are added[a]

Basic ration	Pharmaceutical feed additive	Amount	Time period
Pregestation, breeding, gestation	ASP-250	250 gm/ton	The four weeks after buying new gilts
Pregestation, breeding, gestation	furazolidone	150 gm/ton	The three weeks prior to breeding and the one week prior to farrowing
Pregestation, breeding, gestation	ASP-250	250 gm/ton	The four weeks prior to introducing raised gilts into breeding herd
Lactation	furazolidone	150 gm/ton	The two weeks after farrowing
Starter	tylosin	75 gm/ton	Until pigs reach 40 pounds
Finishing	tylosin	20 gm/ton	From 40 pounds to market

[a]Source: Life Cycle Swine Nutrition, 29, pp. 8-10.

net returns of the production activities, c_j. After finding values for these parameters, the linear program can be set up. By using the assumptions made earlier and additional information and equations to be revealed in this section, values can easily be found for the parameters of the linear program of the swine farm.

Assumption 6 discussed the availability of fixed inputs for use in the production output. The number of hours available in each month, shown in table 4.2a, the capacity for farrowing sows in each farrowing,

Table 4.5a. Price assumptions for variable inputs: Model I

Variable input	Price
Corn	$2.20/bu.
Soybean oilmeal	.12/lb.
Dicalcium phosphate	.10/lb.
Limestone	.02/lb.
Salt	.025/lb.
Trace mineral premix	.10/lb.
Vitamin premix	.60/lb.
Dried whey	.09/lb.
ASP-250	.033/gm
Furazolidone	.06/gm
Tylosin	.12/gm
Group #1 Gilts	
Purchased	100/head
Raised	55.49/head
Group #2 Gilts	
Purchased	124/head
Raised	56.49/head
Group #3 Gilts	
Purchased	149/head
Raised	55.50/head
Group #4 Gilts	
Purchased	208.25/head
Raised	55.50/head
Boars	270/head
Group #1 Feeder pigs	29.54/head
Group #2 Feeder pigs	31.11/head
Group #3 Feeder pigs	30.90/head
Group #4 Feeder pigs	25.26/head
Transportation for:	
Purchased gilts	5/head
Boars	5/head
Purchased feeder pigs	1/head
Market hogs	2/cwt.
Non-conceived gilts	2/cwt.
Culled and market sows	2/cwt.
Market boars	2/cwt.

shown in table 4.2b, the capacity for finishing market hogs, shown in table 4.2c, and the number of boars available, shown in table 4.2d, are all levels of fixed inputs a_{io}, or in other terms, right-hand side (RHS) values of the linear program.

Table 4.5b. Price assumptions for the farm firm's output: Model I

Farm firm output	Price
180 pound Group #1 Market hog	$42.67/cwt.
200 pound Group #1 Market hog	43.31/cwt.
220 pound Group #1 Market hog	45.56/cwt.
240 pound Group #1 Market hog	43.19/cwt.
260 pound Group #1 Market hog	42.74/cwt.
180 pound Group #2 Market hog	41.78/cwt.
200 pound Group #2 Market hog	42.51/cwt.
220 pound Group #2 Market hog	41.96/cwt.
240 pound Group #2 Market hog	42.15/cwt.
260 pound Group #2 Market hog	42.13/cwt.
180 pound Group #3 Market hog	33.32/cwt.
200 pound Group #3 Market hog	32.80/cwt.
220 pound Group #3 Market hog	31.43/cwt.
240 pound Group #3 Market hog	29.91/cwt.
260 pound Group #3 Market hog	27.83/cwt.
180 pound Group #4 Market hog	38.98/cwt.
200 pound Group #4 Market hog	37.94/cwt.
220 pound Group #4 Market hog	37.05/cwt.
240 pound Group #4 Market hog	37.61/cwt.
260 pound Group #4 Market hog	40.19/cwt.
Non-conceived Group #1 Gilts	27.68/cwt.
Non-conceived Group #2 Gilts	37.51/cwt.
Non-conceived Group #3 Gilts	39.77/cwt.
Non-conceived Group #4 Gilts	40.88/cwt.
Culled Group #1 Sows	35.10/cwt.
Culled Group #2 Sows	43.69/cwt.
Market Group #1 Sows	34.41/cwt.
Market Group #2 Sows	35.08/cwt.
Market Boars	32.00/cwt.

By finding additional information on input-output relationships of fixed inputs, production coefficients of fixed inputs, a_{ij}, can be found. Assumption 6 discussed four types of fixed inputs: 1) labor, 2) farrowing capacity, 3) finishing capacity, and 4) boar availability. From this, it is known that at least four types of production coefficients of fixed inputs will be needed.

Additional information needed in forming the production coefficients of fixed labor inputs is the amount of labor that is needed to produce a unit of output. Specific production coefficients of fixed labor inputs indicate the amount of labor in each month needed to perform an activity for one unit of output. Certain labor production coefficients of fixed inputs can be seen in table 4.6a.

Table 4.6a. Certain production coefficients of fixed labor inputs of the linear program of the swine farm: Model I

	Activity						
	A21	A24	A27	A28	A29	A30	A31
Month	Nov. farrow	Feed Aug. pigs to 40 pounds	Feed May pigs to 180 pounds	Feed May pigs to 200 pounds	Feed May pigs to 220 pounds	Feed May pigs to 240 pounds	Feed May pigs to 260 pounds
July 1973	.79		.2	.2	.2	.2	.2
August 1973	.60		.14	.14	.14	.14	.14
September 1973	.52	.33	.15	.15	.15	.15	.15
October 1973	1.90			.04	.08	.15	.15
November 1973	3.17						.04

Each a_{ij} coefficient in table 4.6a indicates the number of hours in each respective month that is needed to perform the activity for each sow or pig. The total hours needed to perform each activity can be found in the Midwest Farm Planning Manual [22]. The breakdown by month can be done once the time period of the activity is known. The time periods for feeding pigs to different market weights are found through the assumed average daily gains for different market weights shown in table 4.1.

Each column of the tableau, shown in table 4.6a, is a subvector of each respective A_j vector of the linear program. The A_j vector of the linear program was defined in section III.D.2 on page 45, following its use in equation 3.10. The same will be true in tables 4.6b, 4.6d, 4.6e, and 4.7.

Additional information needed in forming the production coefficients of fixed farrowing area inputs is the area that is needed for one sow when farrowing. Since the capacity or RHS value is in terms of number of sows, it is obvious that one sow would need one sow area. The production coefficients of fixed farrowing area inputs can be seen in table 4.6b.

Table 4.6b. Production coefficients of fixed farrowing area inputs of the linear program of the swine farm: Model I

	Activity			
	A19	A20	A21	A22
Farrowing capacity	May farrow	Aug. farrow	Nov. farrow	Feb. farrow
May farrow capacity	1			
August farrow capacity		1		
November farrow capacity			1	
February farrow capacity				1

Each a_{ij} coefficient in table 4.6b indicates that each sow farrowed in the farrowing activities requires one unit of farrowing capacity in the respective month in which it farrows.

Additional information needed in forming the production coefficients of fixed finishing area inputs is the area that is needed by the market hog. The area that is needed by a market hog is dependent upon the size of the market hog and the season of the year. Obviously, the larger the market hog, the more area that is needed, but also, the warmer the temperature, the more area that is needed by the market hog. The area that is needed by a market hog depending on its size and the season of the year can be seen in table 4.6c.

Table 4.6c. Finishing area needed by market hog by size and season

Market hog size	Season		
	Winter	Summer	Fall and Spring
180 pounds	10.0 sq. ft.	12.0 sq. ft.	11.0 sq. ft.
200 pounds	10.5 sq. ft.	12.5 sq. ft.	11.5 sq. ft.
220 pounds	11.0 sq. ft.	13.0 sq. ft.	12.0 sq. ft.
240 pounds	11.5 sq. ft.	13.5 sq. ft.	12.5 sq. ft.
260 pounds	12.0 sq. ft.	14.0 sq. ft.	13.0 sq. ft.

Each value in table 4.6c is an a_{ij} coefficient in the linear program of the swine farm firm if the conditions of size and season fit the activity. The values in table 4.6c can be seen as actual a_{ij} values of the linear program of the swine farm in table 4.6d.

Each a_{ij} coefficient in table 4.6d indicates the area in square feet that is needed by the market hog under the activities of feeding market hogs to different market weights during different seasons.

From table 4.2 it can be seen that the number of boars available is two. Two boars are made available only if two boars are purchased. Thus, the linear program of the swine farm must make sure two boars are available. This is done by forcing two boars to be purchased by setting the RHS value of boar availability constraint equal to two boars. Table 4.6e demonstrates this process.

Table 4.6d. Production coefficients of fixed finishing area inputs of the linear program of the swine farm: Model I

	Activity									
	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36
Finishing capacity	Feed May pigs to 180 pounds	Feed May pigs to 200 pounds	Feed May pigs to 220 pounds	Feed May pigs to 240 pounds	Feed May pigs to 260 pounds	Feed Aug. pigs to 180 pounds	Feed Aug. pigs to 200 pounds	Feed Aug. pigs to 220 pounds	Feed Aug. pigs to 240 pounds	Feed Aug. pigs to 260 pounds
Finishing capacity for May pigs	12.0	12.0	12.0	12.5	13.0					
Finishing capacity for August pigs						11.0	11.0	11.0	11.5	12.0
Finishing capacity for November pigs										
Finishing capacity for February pigs										

Table 4.6d. Continued

Finishing capacity	Activity A37	A38	A39	A40	A41	A42	A43	A44	A45	A46
	Feed Nov. pigs to 180 pounds	Feed Nov. pigs to 200 pounds	Feed Nov. pigs to 220 pounds	Feed Nov. pigs to 240 pounds	Feed Nov. pigs to 260 pounds	Feed Feb. pigs to 180 pounds	Feed Feb. pigs to 200 pounds	Feed Feb. pigs to 220 pounds	Feed Feb. pigs to 240 pounds	Feed Feb. pigs to 260 pounds
Finishing capacity for May pigs										
Finishing capacity for August pigs										
Finishing capacity for November pigs	10.0	11.5	12.0	12.5	13.0					
Finishing capacity for February pigs						12.0	12.5	13.0	13.5	14.0

Table 4.6e. Forcing a boar purchasing activity to purchase two boars and the a_{ij} coefficient used: Model I

	RHS	Row type	Activity A05 Boar purchasing
Boar purchasing equality	2	E	+1

Because the row type is an equality (E), two boars will be forced to be purchased. The a_{ij} coefficient has a value of 1.0 since one boar is available when one boar is purchased in the boar purchasing activity.

In addition to the four types of production coefficients for fixed inputs discussed previously, another type of production coefficient that will be used is a production coefficient for use in transfer rows.

Transfer rows have a definite use in any linear program that has activities that occur sequentially. Transfer rows are accounting constraints that keep track of outputs of activities throughout the linear program and thereby give the linear program structure. An output of one activity may be an input in another. Transfer rows transfer the output of one activity to other activities. One transfer row is needed for the transfer of every type of output (input) that must be made in the linear program. Some transfer rows of the linear program can be seen in table 4.7.

Two transfer rows are needed in transferring May pigs from weaning to feeding to market weight. This is because there is an intermediate activity of feeding May pigs to 40 pounds before feeding

Table 4.7. Developing transfer rows (accounting rows)

			Activity						
			A19	A23	A27	A28	A29	A30	A31
Transfer rows	RHS	Row type	May farrow	Feed May pigs to 40 pounds	Feed May pigs to 180 pounds	Feed May pigs to 200 pounds	Feed May pigs to 220 pounds	Feed May pigs to 240 pounds	Feed May pigs to 260 pounds
May weaned pig transfer-row	0	LTE	-7.2	+1					
40 pound May pig transfer-row	0	LTE		-.99	+1	+1	+1	+1	+1

May pigs to market weight. The production coefficients for the transfer rows can be seen in table 4.7.

Notice, the production coefficients carry values that vary in sign and value. The -7.2 value under activity A19 means that the May farrowing activity had an output of 7.2 weaned pigs per farrowing and therefore supplies the May weaned pig transfer row with 7.2 weaned pigs. The +1 value under activity A23 means that the "feed May pigs to 40 pounds" activity requires one weaned May pig in order to feed a May pig to 40 pounds. The -.99 value under activity A23 means that the "feed May pigs to 40 pounds" activity had an output of .99 pigs for every pig fed to 40 pounds and therefore supplies the 40 pound May pig transfer row with 40 pound pigs. (-.99 is used instead of -1 since it is assumed that there is a 1 percent death loss in feeding weaned pigs to 40 pounds.) The +1 value under each activity, A27 through A31, means that each activity of feeding May pigs requires one 40 pound May pig in order to feed one May pig to a market weight. The 0 values (a_{io}) under the RHS column indicate that there must be an ending balance of transfers of zero. This is because the purpose of the transfer row is to transfer output (input) of activities.

Table 4.7 also shows that the row type of the transfer rows is "LTE" or "less than or equal." As a result, the constraints are inequations and are written as inequation 3.6(1) in section III.D.1 and shown as

$$x_{23} - 7.2\, x_{19} \leq 0$$

for the May weaned pig transfer row and

$$\sum_{j=27}^{31} x_j - .99\ x_{23} \leq 0$$

for the 40 pound May pig transfer row.

From the standpoint of the purpose of transfer rows (as accounting constraints transferring output of one activity to other activities), it would seem that the transfer rows would be equalities. From the programming standpoint of the linear program, though, the transfer rows are generally less than or equal constraints.

In section III.D.2, it was shown that in order to solve a linear program, it was necessary to add both slack and artificial variables to the equality constraints and slack variables to the less than or equal constraints. Now, suppose the transfer rows were equality constraints. Both slack and artificial variables must be added to the transfer rows in order to solve the linear program, increasing the size of the linear program (i.e., number of rows and columns) substantially. Now, suppose the transfer rows were less than or equal constraints. It can be seen that by having to add only slack variables to the transfer rows in solving the linear program, the size of the linear program would not increase as much, thus reducing the programming cost in obtaining a solution.

A question may be raised, though, as to whether all of the output of one activity will be transferred to other activities with less than or equal transfer rows. Equality transfer rows assures this to happen. Because of the nature of the linear program (i.e., maximization), all of the output will be transferred with less than or equal transfer rows, also. By maximizing returns of the farm, the firm will not produce any output that will not generate returns. For any output to generate returns to the farm, it must go through all processes (activities) and then to the final activity of marketing. Thus, the marketing activity of each output "pulls" all of the output of each activity through the linear program so that returns may be generated and maximized.

Using equations 3.2 and 3.3, net returns of the production activities c_j may be found. Equation 3.2 stated

$$3.2 \qquad p_j - \sum_k r_k q_{kj} = c_j$$

where p_j is the price received for one unit of output produced by the j-th activity

r_k is the purchase price of the k-th variable input

q_{kj} is the production coefficient of variable inputs which gives the quantity used of the k-th variable input in the production of one unit of output under the j-th activity

c_j is the net return received by producing and selling one unit of output under the j-th activity

Equation 3.3 stated

$$3.3 \qquad p_j - \sum_k V_{kj} = c_j$$

where $V_{kj} = r_k q_{kj}$ and where the new variable V_{kj} is defined as the cost of the k-th variable input and where $\sum_k V_{kj}$ is the total cost of K variable inputs used in producing one unit of output by the j-th activity.

For some variable inputs, e.g., feed, both g_{kj} and r_k are easily obtained. For others, e.g., veterinarian and medicine inputs or fuel and power, the q_{kj} and r_k are difficult to obtain, but V_{kj} (which is equal to $r_k \times q_{kj}$) is easy to obtain.

In actual practice, the c_j are usually computed from

4.0 $$c_j = p_j - \sum_{k'} r_{k'} q_{k'j} - \sum_{k''} V_{k''j}$$

where k' indicates variable inputs for which values of r_k and q_{kj} are known

k'' indicates variable inputs for which values of V_{kj} are known, but where values of r_k and q_{kj} are not known separately

Through Assumption 8, purchase prices of the variable inputs are known and prices received for outputs are known. But, how can q_{kj} be found so as to use equation 4.0 to find c_j, the net return received by producing and selling one unit of output under the j-th activity?

The production coefficient of variable feed inputs can be found by using one of two equations. Equation 4.1 can be used to find production coefficients of variable feed inputs for those activities in which 40 pound pigs are fed to market weight.

4.1 $$q_{kj} = FE \times TG \times \frac{VFI}{2000}$$

where FE is feed efficiency assumed for each market weight under Assumption 5

TG is total gain made by the market hog under the activity

$\frac{VFI}{2000}$ is the pounds of variable feed input in a 2000 pound ration divided by 2000 so as to give the percent of variable feed input in a 2000 pound ration

Equation 4.2 can also be used to find production coefficients of variable feed inputs for those activities in which swine are fed for market.

$$4.2 \qquad q_{kj} = C \times D \times \frac{VFI}{2000}$$

where C is consumption of variable feed inputs by one head of swine per day

D is the number of days in which the swine consumes variable feed inputs

$\frac{VFI}{2000}$ is defined as before

Consumption of variable feed inputs by swine depends upon the type of swine and the season of the year. Consumption by certain types of swine according to season can be seen in table 4.8.

The actual process of finding c_j coefficients may better be described by finding c_j's for various activities of the linear program of the swine farm. Tables 4.9a through 4.9d demonstrate finding c_j values for selected activities. In these tables and the remaining thesis c_j and C_j have the same meaning and are used interchangeably.

Table 4.8. Consumption of variable feed inputs by certain types of swine[a]

Type of swine	Season: Summer	Season: Winter
Open gilts	4-6 lbs./day	5-7 lbs./day
Gestating gilts	4-5 lbs./day	5-6 lbs./day
Lactating gilts	10-12 lbs./day	12-14 lbs./day
Gestating sows	3-4 lbs./day	4-5 lbs./day
Lactating sows	11-13 lbs./day	13-15 lbs./day
Weaned pigs	1-2 lbs./day	1-2 lbs./day
Boar (breeding)	4-6 lbs./day	6-8 lbs./day
Boar (idle)	3-4 lbs./day	4-5 lbs./day

[a]Source: Feeding and Managing the Swine Breeding Herd, 6, p. 6.

Table 4.9a. Finding a C_1 coefficient for activity A01 - Buying Group #1 Gilts: Model I

Input	q_{k1}	r_k[a]	V_{k1}	P_1	C_1
Purchased gilt	1	$100	$100		
Transportation	1	5	5		
Total (Σ)			$105	$0	$-105

[a]From table 4.5a, Price assumptions for variable inputs: Model I.

Each of the four activities that raise gilts represent the last four weeks of the time period it takes to raise a gilt prior to its entering the swine breeding herd. The $56.49 shown in table 4.9b, row 1, represents costs incurred in raising a gilt prior to the last four weeks of the total period it takes to raise a gilt.

In Assumption 4, Discounting to present value, it was decided that discounting was indeed necessary and that the rate of discount would be 12 percent per annum or 1 percent per month. The demonstrations

Table 4.9b. Finding a C_{11} coefficient for activity A11 - Raising Group #1 Gilts: Model I

Input	C^a	D	$\frac{VFI}{2000}$ [b]	q_{k11}	r_k [c]	V_{k11}	P_{11}	C_{11}
Raised gilt				1	56.49	56.49		
Veterinary and medicine						.57		
Fuel and power						.29		
Corn	6 lbs./day	28 days	$\frac{32.732\ \text{bu.}}{2000\ \text{lbs.}}$	2.749 bu.	2.20	6.05		
Soybean oilmeal	6 lbs./day	28 days	$\frac{100\ \text{lbs.}}{2000\ \text{lbs.}}$	8.4 lbs.	.12	1.01		
Limestone	6 lbs./day	28 days	$\frac{15\ \text{lbs.}}{2000\ \text{lbs.}}$	1.26 lbs.	.02	.03		
Dicalcium phosphate	6 lbs./day	28 days	$\frac{30\ \text{lbs.}}{2000\ \text{lbs.}}$	2.52 lbs.	.10	.25		
Salt	6 lbs./day	28 days	$\frac{10\ \text{lbs.}}{2000\ \text{lbs.}}$	.84 lbs.	.025	.02		
Trace mineral premix	6 lbs./day	28 days	$\frac{2\ \text{lbs.}}{2000\ \text{lbs.}}$	.168 lbs.	.10	.02		
Vitamin premix	6 lbs./day	28 days	$\frac{10\ \text{lbs.}}{2000\ \text{lbs.}}$	.84 lbs.	.60	.50		
ASP-250	6 lbs./day	28 days	$\frac{250\ \text{gm}}{2000\ \text{lbs.}}$	21 gm	.033	.69		
Total (Σ)						65.92	0	-65.92

[a] From table 4.8, Consumption of variable feed inputs by certain types of swine.

[b] From table 4.3a, Basic ration for pregestation, breeding, and gestation.

[c] From table 4.5a, Price assumptions for variable inputs: Model I.

Table 4.9c. Finding a C_{27} coefficient for activity A27 - Feeding May pigs to 180 pounds: Model I

Input	FE[a]	TG	$\frac{VFI}{2000}$[b]	q_{k27}	r_k[c]	V_{k27}	P_{27}	C_{27}
Veterinary and medicine						$ 1.90		
Power and fuel						1.00		
Miscellaneous						.20		
Corn	3.4143	140 lbs.	$\frac{\text{29.182 bu.}}{\text{2000 lbs.}}$	6.9744 bu.	$2.20	15.34		
Soybean oilmeal	3.4143	140 lbs.	$\frac{\text{305.81 lbs.}}{\text{2000 lbs.}}$	73.0885 lbs.	.12	8.77		
Limestone	3.4143	140 lbs.	$\frac{\text{15 lbs.}}{\text{2000 lbs.}}$	3.585 lbs.	.02	.07		
Dicalcium phosphate	3.4143	140 lbs.	$\frac{\text{23 lbs.}}{\text{2000 lbs.}}$	5.497 lbs.	.10	.55		
Salt	3.4143	140 lbs.	$\frac{\text{10 lbs.}}{\text{2000 lbs.}}$	2.39 lbs.	.025	.06		
Trace mineral premix	3.4143	140 lbs.	$\frac{\text{2 lbs.}}{\text{2000 lbs.}}$	.478 lbs.	.10	.05		
Vitamin premix	3.4143	140 lbs.	$\frac{\text{10 lbs.}}{\text{2000 lbs.}}$	2.39 lbs.	.60	1.43		
Tylosin	3.4143	140 lbs.	$\frac{\text{20 gm}}{\text{2000 lbs.}}$	4.78 gm	.12	.57		
Total (Σ)						29.94	0	-29.94

[a] From table 4.1, Assumed phenotypic measures of market hogs.

[b] From table 4.3d, Basic finishing ration (14 percent protein).

[c] From table 4.5a, Price assumptions for variable inputs: Model I.

Table 4.9d. Finding a C_{47} coefficient for activity A47 - Marketing 180 pound market hogs: Model I

Input	q_{k47}	r_k[a]	V_{k47}	P_{47}[b]	C_{47}
Transportation	1	\$2/cwt.	\$2/cwt.		
Total			\$2/cwt.	\$42.67/cwt.	\$40.67/cwt.

[a]From table 4.5a, Price assumptions for variable inputs: Model I.

[b]From table 4.5b, Price assumptions for the farm firm's output: Model I.

of finding c_j values for activities in the empirical linear program of the swine farm, shown in tables 4.9a through 4.9d, do not include the discounting procedure so that the c_j value shown in each of the tables still cannot be used as the true c_j value of the activities of the empirical linear program.

By discounting the c_j value in each table, 4.9a through 4.9d, the true c_j value of the activities of the empirical linear program can be found. One formula that may be used in discounting c_j values is shown as

$$4.3 \qquad \hat{c}_j = c_j \left(\frac{1}{(1+i)}\right)^n$$

where $\hat{c}_j$ is the discounted c_j value used in the empirical linear program of the swine farm

c_j is the net revenue of the j-th activity prior to being discounted

i is the rate of discount

n is the number of periods that the c_j value must be discounted

Because the activities of the empirical linear program are broken into months, the c_j values will be discounted by months. Also, since the 22 month period of the empirical linear program begins November 1, 1972, all c_j values of the activities will be discounted to present value as of November 1, 1972.

In order to fully demonstrate the process in finding c_j values, including discounting them to present value; using c_j values of tables 4.9a through 4.9d, the discounting process is shown in table 4.10.

Table 4.10. Discounting c_j values to present value: Model I

Activity	c_j		i	n	$(\frac{1}{1+.01})^n$		$\hat{c}_j$
A01	$- 105		.01	0	0		$- 105
A11	-65.92		.01	0	0		-65.92
A27[a]	-9.98		.01	8	.9227	-9.21	
	-9.98		.01	9	.9135	-9.12	
	-9.98		.01	10	.9044	-9.03	
		-29.94					-27.36
A47	40.67		.01	11	.895336		36.41

[a]Activity A27 lasts for a three-month period and therefore each month's net revenue must be discounted separately. Each month's net revenue may be totaled after being discounted to November 1, 1975, present value.

4. Specific description of the linear program of the swine farm

Up until now, the material presented has been general to give a feeling of the process of developing a linear program and also to give an idea of what the swine farm in the empirical linear program is like. Unfortunately, due to the size of the linear program, it is impractical to specifically describe the total linear program of the swine farm here. Instead, portions of the linear program will be described. (Persons who are interested in the total empirical linear program may obtain the linear program from Dr. George Ladd, 478D East Hall, Department of Economics, Iowa State University, Ames, Iowa, 50011.)

Figure 4.1a divides the total linear program of the swine farm into areas. Area I represents the RHS values (a_{io}) of all the constraint equations. In reporting linear program tableaus (shown in figure 4.2) the RHS values are ironically placed on the left-hand side of the tableau. These values, though, are still termed RHS values because they are shown on the right hand side of constraints, as shown by constraints in section III.B.

Area II represents the type of constraint equations that are within the linear program; whether they be less than or equal, greater than or equal, or equality constraints. Area III represents c_j and a_{ij} coefficients of activities that occur within the swine farm, independent or semi-independent of the number of farrowings that take place during the time period of the linear program. Area IV represents c_j and a_{ij} coefficients of activities relating to the first farrowing of the swine farm. Area V represents c_j and a_{ij} coefficients of activities relating to the second farrowing of the swine farm. Areas VI and VII represent c_j and a_{ij} coefficients of activities relating to the third and fourth farrowings of the swine farm, respectively.

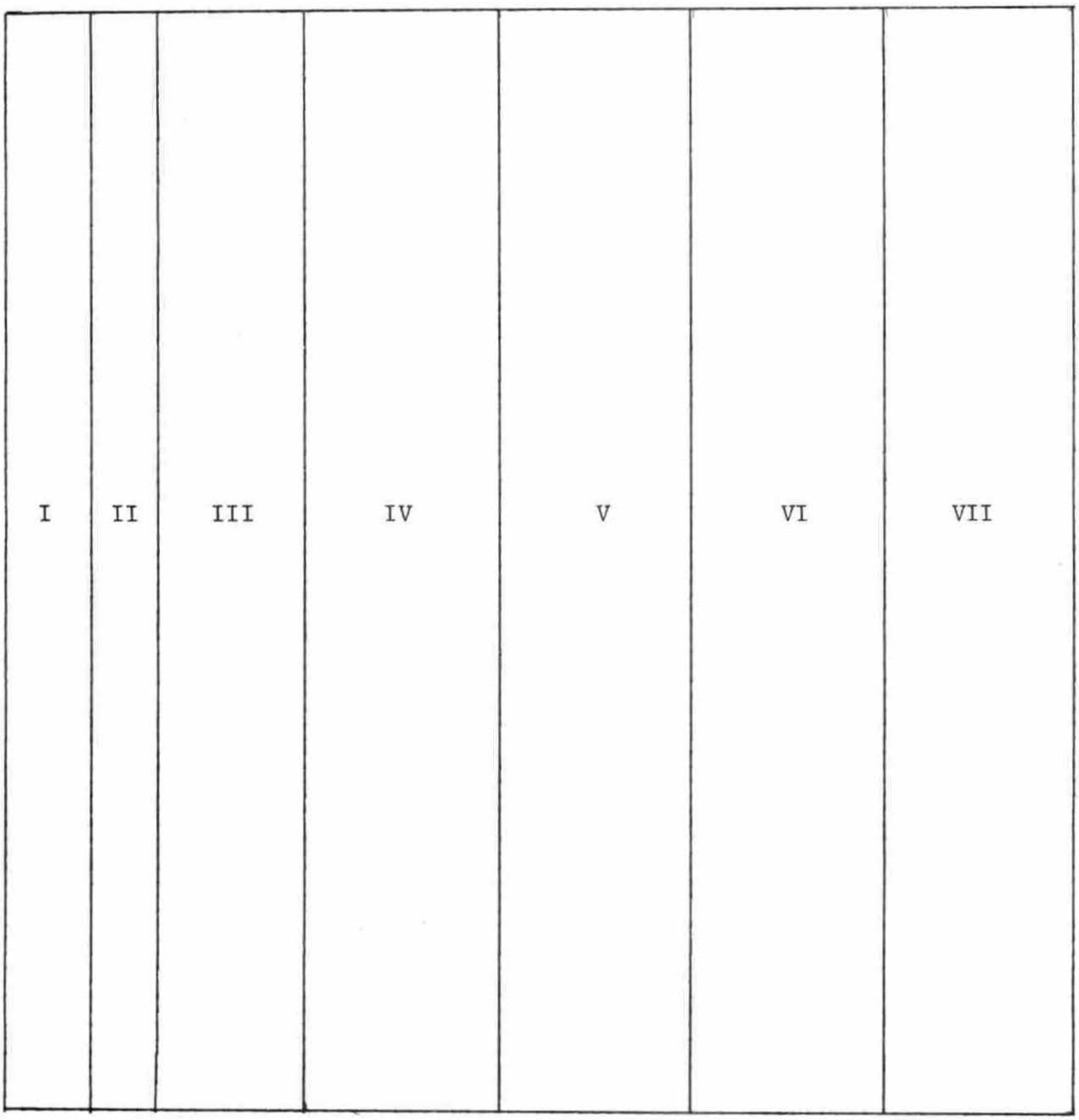

Figure 4.1a. Empirical linear program of the swine farm by areas: Model I

Each area of the linear program shown in figure 4.1a may be broken into sections as shown in figure 4.1b. In figure 4.1b, each section and its dimension or the number of rows by the number of columns (shown under the section letter) are shown.

Figure 4.1b is simply a restatement of the typical linear program shown in section III.D.1 by equation 3.5 and constraints 3.6. Sections O', Q, R, S, and T contain c_j of equation 3.5. The RHS column (area I of figure 4.1a contains a_{io} of constraints 3.6. The second column (area II of figure 4.1a) identifies the direction of the inequality constraints. "LTE" represents less than or equal constraints. "EQ" represents equality constraints. The remaining sections contain a_{ij}. One column of figure 4.1b contains $\binom{c_j}{A_j}$. One row contains one constraint from constraints 3.6 in section III.D.1.

In order that the linear program of the swine farm be more specifically described, certain sections of areas I, II, and III are shown in greater detail. Figure 4.2 shows sections RHS1, RHS4, RHS5, O', A', D', and I' in detail. (Sections RHS2, RHS3, RHS6, B', C', and N' are not shown in detail since each has coefficient values of zero.)

Sections A', B', C', D', I', and N' of figure 4.1b contain a_{ij} coefficients of activities that occur within the swine farm, independent and semi-independent of the number of farrowings that take place during the time period of the linear program. Looking at figure 4.2, the specific activities can be seen. Figure 4.2 also gives the RHS values of each constraint that has an a_{ij} coefficient in at least one of the activities and gives the name and type of constraint.

		O' (1x7)	Q (1x18)	R (1x18)	S (1x18)	T (1x18)
RHS 1 (22x1)	LTE	A' (22x7)	A (22x18)	B (22x18)	C (22x18)	D (22x18)
RHS 2 (4x1)	LTE	B' (4x7)	E (4x18)	F (4x18)	G (4x18)	H (4x18)
RHS 3 (4x1)	LTE	C' (4x7)	I (4x18)	J (4x18)	K (4x18)	L (4x18)
RHS 4 (3x1)	EQ	D' (3x7)	E' (3x18)	F' (3x18)	G' (3x18)	H' (3x18)
RHS 5 (4x1)	LTE	I' (4x7)	J' (4x18)	K' (4x18)	L' (4x18)	M' (4x18)
RHS 6 (44x1)	LTE	N' N'1 (11x7)	M M1 (11x18)	N N1 (11x18)	O O1 (11x18)	P P1 (11x18)
		N'2 (11x7)	M2 (11x18)	N2 (11x18)	O2 (11x18)	P2 (11x18)
		N'3 (11x7)	M3 (11x18)	N3 (11x18)	O3 (11x18)	P3 (11x18)
		N'4 (11x7)	M4 (11x18)	N4 (11x18)	O4 (11x18)	P4 (11x18)

Figure 4.1b. Empirical linear program of the swine farm by sections: Model I

Row name	RHS	Row type	A05	A10	A71	A72	A73	A74	A75
C-row			-275	-101.91	28.44	28.17	26.59	30.85	37.70
M01	160	LTE	+.1	+.12					
M02	196	LTE		+.12					
M03	216	LTE		+.12					
M04	192	LTE		+.12					
M05	198	LTE		+.12					
M06	160	LTE		+.12					
M07	160	LTE		+.12					
M08	160	LTE		+.12				+.04	
M09	216	LTE		+.12					
M10	208	LTE		+.12					
M11	168	LTE		+.12					+.04
M12	160	LTE		+.12					
M13	160	LTE					+.03		
M14	196	LTE			+.03				
M15	216	LTE							
M16	192	LTE							
M17	198	LTE				+.03			
M18	160	LTE							
M19	160	LTE							
M20	160	LTE							
M21	216	LTE							
M22	208	LTE							
F01-F04	25	LTE				B'			
R01-R04	3250	LTE				C'			
R25	2	EQ	+1						
R36	0	EQ	-1	+1					
R37	0	EQ		-4			+1		
R34	0	LTE			+1				
R35	0	LTE				+1			
R42	0	LTE						+1	
R43	0	LTE							+1
R21-R24 R26-R33 R38-R41 R46-R73		LTE				N'			

Figure 4.2. Sections RHS1, RHS4, RHS5, O', A', D', and I' from figure 4.1b

The following provides a specific description of the C-row coefficients, constraints, activities, and coefficients shown in figure 4.2.

C-row coefficients

The C-row consists of net return coefficients from each of the activities. The derivation of the c_j values has been previously discussed and demonstrated in section IV.A.1.c.

Constraints

M01-M22

These are monthly labor constraints for the months, November 1, 1972, through August 31, 1974, respectively. The RHS values are maximum labor hours available in each month. The constraints are therefore less than or equal constraints.

F01-F04

These are farrowing constraints for farrowing during May, August, November, and February, respectively. These constraints set the maximum number of sows that can be farrowed at one farrowing. The RHS for each constraint is set at 25 sows. The RHS values are maximum values and the constraints are therefore less than or equal constraints.

R01-R04

These are finishing area constraints for the four groups of market hogs that are fed during the time period of the linear program. These constraints set the maximum area in square feet that is available to finish market hogs. The RHS for each constraint is 3,250 square feet. The RHS values are maximum values and the constraints are therefore less than or equal constraints.

R25

The RHS of this equality constraint is 2. This forces the purchase of two boars in order to breed the 25 females that may farrow at each of the farrowing times.

R36

This is an equality transfer row that moves the purchased boar to a feeding and breeding activity. This transfer row must be an equality so as to force the purchased boar into the feeding and breeding activity. The RHS value, as for all transfer rows, is zero.

R37

This is an equality transfer row that transfers the boar, after he has served his purpose, into a marketing activity. The boar is transferred by hundredweight since he is marketed by hundredweight. The RHS value is zero.

R34 and R35

These are transfer rows that transfer sows farrowed in November and February into marketing activities occurring in early December and early March, respectively. The sows are transferred by hundredweight, since they are marketed by hundredweight. The RHS value is zero and the row type is less than or equal.

R42 and R43

These are transfer rows that transfer gilts that farrow in May and August into marketing activities occurring in early May and early August, respectively. These gilts are marketed because they are culled from the breeding herd. The gilts are transferred by hundredweight, because they are marketed by hundredweight. The RHS value is zero and the row type is less than or equal.

R21 - R24, R26 - R33, R38 - R41, R46 - R73

These are transfer rows whose coefficients make up sections N', M, N, O, and P. These transfer rows are not applicable to activities A05, A10, A71, A72, A73, A74, and A75. As a result, sections B', C', and N' have a_{ij} coefficients of zero. Transfer rows R21 - R24, R26 - R33, R38 - R41, and R46 - R73 will be discussed later, though.

Activities

A05

This is a buying activity for the purpose of buying boars. Boars are purchased by the head. The C_5 value, \$-275, is the discounted negative variable cost of buying one boar. The C_5 value is negative since no returns are generated by purchasing a boar; only costs are generated. This same case was alluded to earlier in section III.C, page 35.

a_{ij} coefficients:

+.1 in M01 means that .1 hours of November, 1972, labor is used when one boar is purchased during November 1972.

+1 in R25 means that for every boar that is forced to be purchased, one boar is purchased.

-1 in R36 means that when one boar is purchased, one boar is supplied to the boar transfer row (R36) to be transferred into a boar feeding activity (A10).

A10

This is an activity to feed and care for boars. Boars are fed and cared for by the head. The C_{10} value, \$-101.91, is the discounted negative cost of feeding and caring for one boar. The C_{10} value is negative for the same reason as the C_5 value.

a_{ij} coefficients:

+.12 in M01 through M12 means that .12 hours of labor in each respective month is used for each boar that is fed and cared for.

+1 in R36 means that one boar is required for every boar that is fed and cared for in the activity.

-4 in R37 means that 4 hundredweights are supplied to the transfer row (R37) for each boar fed and cared for in the activity so that each boar may be marketed by the hundredweight.

A71

This is an activity to market females that farrowed in November. The females are marketed by the hundredweight. The C_{71} value, \$28.44, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

+.03 in M14 means that .03 hours of December, 1973, labor is required for each hundredweight marketed.

+1 in R34 means that one hundredweight is required by the activity in order to market one hundredweight.

A72

This is an activity to market females that farrowed in February. The females are marketed by the hundredweight. The C_{72} value, \$28.17, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

+.03 in M17 means that .03 hours of March, 1974, labor is required for each hundredweight marketed.

+1 in R35 means that one hundredweight is required by the activity in order to market one hundredweight.

A73

This is an activity to market boars that have served their purpose of breeding females. The boars are marketed by the hundredweight. The C_{73} value, \$26.59, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

+.03 in M13 means that .03 hours of November, 1974, labor is required for each hundredweight marketed.

+1 in R37 means that one hundredweight is required by the activity in order to market one hundredweight.

A74

This is an activity to market gilts that have been culled from the breeding herd following farrowing in May. The culled gilts are marketed by the hundredweight. The C_{74} value, \$30.85, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

+.04 in M08 means that .04 hours of June, 1973, labor is required for each hundredweight marketed.

+1 in R42 means that one hundredweight is required by the activity in order to market one hundredweight.

A75

This is an activity to market gilts that have been culled from the breeding herd following farrowing in August. The culled gilts are marketed by the hundredweight. The C_{75} value, \$37.70, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

+.04 in M11 means that .04 hours of September, 1973, labor is required for each hundredweight marketed.

+1 in R43 means that one hundredweight is required by the activity in order to market one hundredweight.

Figure 4.3 shows sections Q, A, E, I, and part of M in detail. (Sections E' and part of M have a_{ij} coefficients that are zero and are therefore not shown in detail.) Figures 4.1b and 4.3 show that each section, M, N, O, and P, can actually be broken into four subsections, each subsection having an 11 x 18 dimension. This can be done since subsection M1, of section M, has certain non-zero a_{ij} coefficients which correspond with the activities within area IV. The remaining 3 subsections have zero a_{ij} coefficients. Also, subsection N2, of section N, has certain non-zero a_{ij} coefficients which correspond with the activities within area V. Subsection 03, of section O, has certain non-zero a_{ij} coefficients which correspond with

Row name	RHS	Row type	A01	A06	A11	A15	A19	A23	A27	A28
C-row			-105	-11.11	-65.92	-11.86	-51.68	-6.87	-27.36	-31.40
M01	160	LTE	.15		.45					
M02	196	LTE		.70		.44				
M03	216	LTE					.80			
M04	192	LTE					.61			
M05	198	LTE					.53			
M06	160	LTE					1.99			
M07	160	LTE					3.04			
M08	160	LTE						.35		
M09	216	LTE							.2	.2
M10	208	LTE							.14	.14
M11	168	LTE							.15	.15
M12	160	LTE								.04
M13	160	LTE								
M14	196	LTE								
M15	296	LTE								
M16	292	LTE								
M17	198	LTE								
M18	160	LTE								
M19	160	LTE								
M20	160	LTE								
M21	216	LTE								
M22	208	LTE								
F01	25	LTE					+1			
F02	25	LTE								
F03	25	LTE								
F04	25	LTE								

Figure 4.3. Sections RHS1, RHS2, RHS3, RHS5, RHS6, Q, A, E, I, J', and M1 from figure 4.1b

Row name	RHS	Row type	A01	A06	A11	A15	A19	A23	A27	A28
R01	3250	LTE							12.0	12.0
R02	3250	LTE								
R03	3250	LTE								
R04	3250	LTE								
R25										
R36		EQ				E'				
R37										
R34		LTE								
R35		LTE								
R42		LTE					-.033			
R43		LTE								
R21		LTE	-1	+1						
R26		LTE		-.95		-.95	+1			
R30		LTE			-1	+1				
R38		LTE		-.125		-.125				
R46		LTE					-7.2	+1		
R50		LTE						-.99	+1	+1
R54		LTE							-1.782	
R55		LTE								-1.98
R56		LTE								
R57		LTE								
R58		LTE								
R22,R27,R31,R39 R47,R51,R59-R63		LTE				M2				
R32							-.98			
R23,R28,R40 R48,R52,R64-R68		LTE				M3				
R24,R29,R33,R41 R49,R53,R69-R73		LTE				M4				

Figure 4.3. Continued

Row name	RHS	Row type	A29	A30	A31	A47	A48	A49	A50	A51	A67	A76
C-row			-35.57	-39.94	-44.46	36.41	36.99	38.11	36.51	36.11	25.17	-28.47
M01	160	LTE										
M02	196	LTE										
M03	216	LTE									.05	
M04	192	LTE										
M05	198	LTE										
M06	160	LTE										
M07	160	LTE										
M08	160	LTE										.1
M09	216	LTE	.2	.2	.2							
M10	208	LTE	.14	.14	.14							
M11	168	LTE	.15	.15	.15							
M12	160	LTE	.08	.15	.15	.015	.015	.015				
M13	160	LTE			.04				.015	.015		
M14	196	LTE										
M15	296	LTE										
M16	292	LTE										
M17	198	LTE										
M18	160	LTE										
M19	160	LTE										
M20	160	LTE										
M21	216	LTE										
M22	208	LTE										
F01	25	LTE										
F02	25	LTE										
F03	25	LTE										
F04	25	LTE										

Figure 4.3. Continued

Row name	RHS	Row type	A29	A30	A31	A47	A48	A49	A50	A51	A67	A76
R01	3250	LTE	12.0	12.5	13.0							
R02	3250	LTE										
R03	3250	LTE										
R04	3250	LTE										
R25												
R36		EQ					E'					
R37												
R34		LTE										
R35		LTE										
R42		LTE										
R43		LTE										
R21		LTE										
R26		LTE										
R30		LTE										
R38		LTE									+1	
R46		LTE										
R50		LTE	+1	+1	+1							-1
R54		LTE				+1						
R55		LTE					+1					
R56		LTE	-2.178					+1				
R57		LTE		-2.376					+1			
R58		LTE			-2.574					+1		
R22,R27,R31,R39 R47,R51,R59-R63		LTE					M2					
R32 R23,R28,R40 R48,R52,R64-R68		LTE					M3					
R24,R29,R33,R41 R49,R53,R69-R73		LTE					M4					

Figure 4.3. Continued

the activities within area VI. Subsection P4, of section P, has certain nonzero a_{ij} coefficients which correspond with the activities within area VII.

The following provides a specific description of the C-row coefficients, constraints, and activities shown in figure 4.3.

C-row coefficients

Again, the C-row consists of net return coefficients from each of the activities. The derivation of the c_j values has been previously discussed and demonstrated in section IV.A.3.

Constraints

M01 - M22, F01 - F04, R01 - R04, R25, R36, R37, R34 - R35, R42 - R43

These constraints were defined earlier in describing figure 4.2.

R21

This is a transfer row. It transfers purchased gilts into an activity that prepares the gilts to be introduced into the breeding herd and also for breeding in order to farrow in May. The gilts are transferred by the head. The RHS value is zero.

R26

This is a transfer row. It transfers both purchased and raised gilts into the May farrowing activity. The gilts are transferred by the head. The RHS value is zero.

R30

This is a transfer row. It transfers gilts that were raised into an activity that prepares them for breeding and farrowing in May. The gilts are transferred by the head. The RHS value is zero.

R38

This is a transfer row. It transfers gilts that did not conceive at breeding for first farrowing to a marketing activity. The gilts are transferred by the hundredweight. The RHS value is zero.

R46

This is a transfer row which transfers weaned pigs farrowed in May into a feeding activity that feeds May pigs to 40 pounds. The pigs are transferred by the head. The RHS value is zero.

R50

This is a transfer row which transfers 40 pound feeder pigs (farrowed in May or purchased in June) to finishing activities that feed hogs to market weight. The 40 pound pigs are transferred by the head. The RHS value is zero.

R54

This is a transfer row which transfers 180 pound market hogs (farrowed in May or purchased in June) to a marketing activity. The market hogs are transferred by hundredweight. The RHS value is zero.

R55

This is a transfer row which transfers 200 pound market hogs (farrowed in May or purchased in June) to a marketing activity. The market hogs are transferred by hundredweight. The RHS value is zero.

R56

This is a transfer row which transfers 220 pound market hogs (farrowed in May or purchased in June) to a marketing activity. The market hogs are transferred by hundredweight. The RHS value is zero.

R57

This is a transfer row that transfers 240 pound market hogs (farrowed in May or purchased in June) to a marketing activity. The market hogs are transferred by hundredweight. The RHS value is zero.

R58

This is a transfer row that transfers 260 pound market hogs (farrowed in May or purchased in June) to a marketing activity. The market hogs are transferred by hundredweight. The RHS value is zero.

Subsections M2 and M4 are each 11 x 18 in dimension and are totally filled with a_{ij} coefficients of zero. Subsection M3 is of 11 x 18 dimension, also, and is totally filled with a_{ij} coefficients of zero with one exception, the a_{ij} coefficient $a_{32,19}$ is nonzero. The constraint R32 parallels constraint R30 in that it transfers gilts into an activity that prepares them for breeding and farrowing. But, since gilts that farrow in May may farrow again in November, they must be prepared for a second breeding and farrowing. The gilts that farrow in May must be transferred into an activity that prepares them for breeding and farrowing in November. This is the purpose of constraint R32 and coefficient $a_{32,19}$; transfer May farrowed gilts into an activity to prepare females for breeding and farrowing in November. Constraint R32 is in subsection M3, though, since it relates to activities centered around the third farrowing (or farrowing in November).

Activities

A01

This is a buying activity for the purpose of buying open gilts with the idea of breeding them so they farrow in May. The gilts are purchased by the head. The C_1 value, \$-105, is the discounted negative variable cost of purchasing one gilt. This value is negative due to zero returns being generated as in activities A5 and A10.

a_{ij} coefficients:

.15 in M01 means that .15 hours of November 1972 labor is used when one gilt is purchased during November 1972.

-1 in R21 means that when one gilt is purchased, one gilt is supplied to the purchased gilt transfer row (R21) to be transferred into an activity to prepare the gilt for introduction into the breeding herd.

A06

This is an activity to prepare newly purchased gilts for introduction into the swine breeding herd so as to farrow in May. This activity lasts four weeks and includes isolation of the gilts, testing for disease organisms, and feeding and observation. Gilts are cared for by the head. The C_6 value, \$-11.11, is the discounted negative variable cost of feeding and caring for one gilt.

a_{ij} coefficients:

.70 in M02 means that .70 hours of December 1972 labor is used when one gilt is fed and cared for during the four week period.

+1 in R21 means that one gilt is required for each gilt that is fed and cared for in the activity.

-.95 in R26 means that .95 of each gilt that is fed and cared for under the activity will be supplied to the transfer row (R26) so as to be transferred to the May farrowing activity. (This is the same as saying 95 percent of the gilts prepared for breeding and farrowing will conceive and be transferred to the May farrowing activity.)

-.125 in R38 means that .125 of each gilt that is fed and cared for under the activity will be supplied to the transfer row (R38) so as to be transferred to a marketing activity (A67). (This is the same as saying 5 percent of the 250 pound gilts prepared for breeding and farrowing will not conceive and be transferred to a marketing activity.)

A11

This is an activity where gilts are raised during the last four week period of the total period it takes to raise gilts. This activity includes feeding and observation of the gilts. Gilts are raised by the head. The C_{11} value, \$-65.92, is the discounted negative variable cost of raising one gilt.

a_{ij} coefficients:

.45 in M01 means that .45 hours of November 1972 labor is used when one gilt is raised during the last four week period of the total rearing period.

-1 in R30 means that one gilt is supplied to the transfer row (R30) so as to be transferred into an activity that prepares raised gilts for breeding and farrowing.

A15

This is an activity where gilts that have been raised from weaning are prepared for breeding and farrowing in May. This activity includes testing for disease organisms, feeding, and observation of the gilts. Gilts are cared for by the head. The C_{15} value, \$-11.86, is the discounted negative variable cost of preparing one gilt for breeding and farrowing in May, 1973.

a_{ij} coefficients:

.44 in M02 means that .44 hours of December 1972 labor is used when one raised gilt is prepared for breeding and farrowing in May.

-.95 in R26 means that .95 of each gilt that is prepared for breeding and farrowing in May will be supplied to the transfer row (R26) so as to be transferred to the May farrowing activity (A19).

+1 in R30 means that for each raised gilt that is prepared for breeding and farrowing in May, one raised gilt is required.

-.125 in R38 means that .125 of each raised gilt that is prepared for breeding and farrowing in May will be supplied to the transfer row (R38) so as to be transferred to a marketing activity (A67).

A19

This activity includes one week of pregestation and breeding, 16 weeks of gestation, and four weeks of farrowing and lactation, occurring during the months of January 1973 through May 1973. The females (gilts) are farrowed by the head. The C_{19} value, \$-51.68, is the discounted negative variable cost of breeding and farrowing one gilt in May 1973.

a_{ij} coefficients:

.80 in M03 means that .80 hours of January 1973 labor is used when one gilt is to be farrowed in May 1973.

.61 in M04 means that .61 hours of February 1973 labor is used when one gilt is to be farrowed in May 1973.

.53 in M05 means that .53 hours of March 1973 labor is used when one gilt is to be farrowed in May 1973.

1.99 in M06 means that 1.99 hours of April 1973 labor is used when one gilt is to be farrowed in May 1973.

3.04 in M07 means that 3.04 hours of May 1973 labor is used when one gilt is to be farrowed in May 1973.

+1 in F01 means that one gilt utilizes one May farrowing space when farrowing in May 1973.

-.033 in R42 means that .033 hundredweight of gilt is supplied to the transfer row (R42), after being culled for some reason, so as to be transferred to a marketing activity (A74).

+1 in R26 means that one gilt is required by the activity from the transfer row for purchased and raised gilts in order to farrow one gilt in May.

-7.2 in R46 means that 7.2 weaned pigs are supplied to the transfer row (R46) for each gilt farrowed in May.

-.98 in R32 means that .98 of a gilt is supplied to the transfer row (R30) to be transferred to an activity to be prepared for a second farrowing in November for each gilt farrowed in May.

A23

This is an activity where weaned pigs, farrowed in May, are fed and cared for until the pigs reach 40 pounds. The activity has a time period of four weeks. The May pigs are raised to 40 pounds per head. The C_{23} value, \$-6.87, is the discounted negative variable cost of raising one weaned pig, farrowed in May, to 40 pounds.

a_{ij} coefficients:

.35 in M08 means that .35 hours of June 1973 labor is used in feeding and caring for one pig farrowed in May from the time it is weaned until it is 40 pounds.

+1 in R46 means that one weaned pig farrowed in May is required to feed and care for one until it reaches 40 pounds.

-.99 in R50 means that .99 of each weaned pig is supplied to the transfer row (R50) for each pig that is fed and cared for until it reaches 40 pounds so that it may be transferred to an activity that feeds the pig to market weight.

A27

This is an activity where 40 pound, May farrowed, pigs are fed to a market weight of 180 pounds. The pigs are fed by the head. The C_{27} value, \$-27.36, is the discounted negative variable cost of feeding one 40 pound pig to 180 pounds in 91.83 days. The 91.83 days is found by dividing the pounds gained (140) by the assumed average daily gain (1.5246) shown in table 4.1.

a_{ij} coefficients:

.2 in M09 means that .2 hours of July 1973 labor is used in feeding and caring for May farrowed market hogs gaining 140 pounds.

.14 in M10 means that .14 hours of August 1973 labor is used in feeding and caring for May farrowed market hogs gaining 140 pounds.

.15 in M11 means that .15 hours of September 1973 labor is used in feeding and caring for May farrowed market hogs gaining 140 pounds.

12.0 in R01 means that each market hog fed to 180 pounds requires 12 square feet of finishing space while being fed to 180 pounds.

+1 in R50 means that one 40 pound feeder pig is required in order to feed and care for one May farrowed, market hog gaining 140 pounds.

-1.782 in R54 means that 1.782 hundredweights are supplied to the transfer row (R54) for each market hog fed to 180 pounds so that 99 percent of the market hogs fed can be marketed. (The other 1 percent of the market hogs is assumed to die.)

A28

This is an activity where 40 pound, May farrowed, pigs are fed to a market weight of 200 pounds. The pigs are fed by the head. The C_{28} value, \$-31.40, is the discounted negative variable cost of feeding one 40 pound pig to 200 pounds in 101.24 days. The 101.24

days is found by dividing the pounds gained (160) by the assumed average daily gain (1.5804) shown in table 4.1.

a_{ij} coefficients:

.2 in M09 means that .2 hours of July 1973 labor is used in feeding and caring for May farrowed market hogs gaining 160 pounds.

.14 in M10 means that .14 hours of August 1973 labor is used in feeding and caring for May farrowed market hogs gaining 160 pounds.

.15 in M11 means that .15 hours of September 1973 labor is used in feeding and caring for May farrowed market hogs gaining 160 pounds.

.04 in M12 means that .04 hours of October 1973 labor is used in feeding and caring for May farrowed market hogs gaining 160 pounds.

12.0 in R01 means that each market hog fed to 200 pounds requires 12 square feet of finishing space while being fed to 200 pounds.

+1 in R50 means that one 40 pound feeder pig is required in order to feed and care for one May farrowed market hog gaining 160 pounds.

-1.98 in R55 means that 1.98 hundredweights are supplied to the transfer row (R55) for each market hog fed to 200 pounds so that 99 percent of the market hogs fed can be marketed. (The other 1 percent is assumed to die.)

A29

This is an activity where 40 pound, May farrowed pigs are fed to a market weight of 220 pounds. The pigs are fed by the head. The C_{29} value, \$-35.57, is the discounted negative variable cost of feeding one 40 pound pig to 220 pounds in 110.44 days. The 110.44 days is found by dividing the pounds gained (180) by the assumed average daily gain (1.6298) shown in table 4.1.

a_{ij} coefficients:

.2 in M09 means that .2 hours of July 1973 labor is used in feeding and caring for May farrowed market hogs gaining 180 pounds.

.14 in M10 means that .14 hours of August 1973 labor is used in feeding and caring for May farrowed market hogs gaining 180 pounds.

.15 in M11 means that .15 hours of September 1973 labor is used in feeding and caring for May farrowed market hogs gaining 180 pounds.

.08 in M12 means that .08 hours of October 1973 labor is used in feeding and caring for May farrowed market hogs gaining 180 pounds.

12.0 in R01 means that each market hog fed to 220 pounds requires 12 square feet of finishing space while being fed to 220 pounds.

+1 in R50 means that one 40 pound feeder pig is required in order to feed and care for one May farrowed market hog gaining 180 pounds.

-2.178 in R56 means that 2.178 hundredweights are supplied to the transfer row (R56) for each market hog fed to 220 pounds so that 99 percent of the market hogs fed can be marketed. (The other 1 percent is assumed to die.)

A30

This is an activity where 40 pound, May farrowed pigs are fed to a market weight of 240 pounds. The pigs are fed by the head. The C_{30} value, \$-39.94, is the discounted negative variable cost of feeding one 40 pound pig to 240 pounds in 119.56 days. The 119.56 days is found by dividing the pounds gained (200) by the assumed average daily gain (1.6728) shown in table 4.1.

a_{ij} coefficients:

.2 in M09 means that .2 hours of July 1973 labor is used in feeding and caring for May farrowed market hogs gaining 200 pounds.

.14 in M10 means that .14 hours of August 1973 labor is used in feeding and caring for May farrowed market hogs gaining 200 pounds.

.15 in M11 means that .15 hours of September 1973 labor is used in feeding and caring for May farrowed market hogs gaining 200 pounds.

.15 in M12 means that .15 hours of October 1973 labor is used in feeding and caring for May farrowed market hogs gaining 200 pounds.

12.5 in R01 means that each market hog fed to 240 pounds requires 12.5 square feet of finishing space while being fed to 240 pounds.

+1 in R50 means that one 40 pound feeder pig is required in order to feed and care for one May farrowed market hog gaining 200 pounds.

-2.376 in R57 means that 2.376 hundredweights are supplied to the transfer row (R57) for each market hog fed to 240 pounds so that 99 percent of the market hogs fed can be marketed. (The other 1 percent is assumed to die.)

A31

This is an activity where 40 pound, May farrowed pigs are fed to a market weight of 260 pounds. The pigs are fed by the head. The C_{31} value, \$-44.46, is the discounted net revenue generated by feeding one 40 pound pig to 260 pounds in 121.59 days. The 121.59 days is found by dividing the pounds gained (220) by the assumed average daily gain (1.7109) shown in table 4.1.

a_{ij} coefficients:

.2 in M09 means that .2 hours of July 1973 labor is used in feeding and caring for May farrowed market hogs gaining 220 pounds.

.14 in M10 means that .14 hours of August 1973 labor is used in feeding and caring for May farrowed market hogs gaining 220 pounds.

.15 in M11 means that .15 hours of September 1973 labor is used in feeding and caring for May farrowed market hogs gaining 220 pounds.

.15 in M12 means that .15 hours of October 1973 labor is used in feeding and caring for May farrowed market hogs gaining 220 pounds.

.04 in M13 means that .04 hours of November 1973 labor is used in feeding and caring for May farrowed market hogs gaining 220 pounds.

13.0 in R01 means that each market hog fed to 260 pounds requires 13.0 square feet of finishing space while being fed to 260 pounds.

+1 in R50 means that one 40 pound feeder pig is required in order to feed and care for one May farrowed market hog gaining 220 pounds.

-2.574 in R58 means that 2.574 hundredweights are supplied to the transfer row (R58) for each market hog fed to 260 pounds so that 99 percent of the market hogs fed can be marketed. (The other 1 percent is assumed to die.)

A47

This is a marketing activity for May farrowed, 180 pound market hogs. The 180 pound market hogs are marketed by the hundredweight. The C_{47} value, \$36.41, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.015 in M12 means that .015 hours of October 1973 labor is used for each hundredweight marketed in marketing a 180 pound market hog.

+1 in R54 means that the activity requires one hundredweight in order to market one hundredweight.

A48

This is a marketing activity for May farrowed, 200 pound market hogs. The 200 pound market hogs are marketed by the hundredweight. The C_{48} value, \$36.99, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.015 in M12 means that .015 hours of October 1973 labor is used for each hundredweight marketed in marketing a 200 pound market hog.

+1 in R55 means that the activity requires one hundredweight in order to market one hundredweight.

A49

This is a marketing activity for May farrowed, 220 pound market hogs. The 220 pound market hogs are marketed by the hundredweight. The C_{49} value, \$38.11, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.015 in M12 means that .015 hours of October 1973 labor is used for each hundredweight marketed in marketing a 220 pound market hog.

+1 in R56 means that the activity requires one hundredweight in order to market one hundredweight.

A50

This is a marketing activity for May farrowed, 240 pound market hogs. The 240 pound market hogs are marketed by the hundredweight. The C_{50} value, \$36.51, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.015 in M13 means that .015 hours of November 1973 labor is used for each hundredweight marketed in marketing a 240 pound market hog.

+1 in R57 means that the activity requires one hundredweight in order to market one hundredweight.

A51

This is a marketing activity for May farrowed, 260 pound market hogs. The 260 pound market hogs are marketed by the hundredweight. The C_{51} value, \$36.11, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.015 in M13 means that .015 hours of November 1973 labor is used for each hundredweight marketed in marketing a 240 pound market hog.

+1 in R58 means that the activity requires one hundredweight in order to market one hundredweight.

A67

This is a marketing activity for gilts that do not conceive in breeding so as to farrow in May. The gilts that do not conceive are marketed by the hundredweight. The C_{67} value, \$25.17, is the discounted net revenue generated by marketing one hundredweight.

a_{ij} coefficients:

.05 in M03 means that .05 hours of January 1973 labor is used for each hundredweight marketed in marketing the gilts that do not conceive.

+1 in R38 means that the activity requires one hundredweight in order to market one hundredweight.

A76

This is a purchasing activity for the purpose of buying May farrowed 40 pound feeder pigs to finish to a marketing weight. The 40 pound feeder pigs are purchased by the head. The C_{76} value, \$-28.47, is the discounted negative variable cost of purchasing one May farrowed 40 pound feeder pig.

a_{ij} coefficients:

.1 in M08 means that .1 hours of June 1973 labor is used when one May farrowed, 40 pound feeder pig is purchased in June 1973.

-1 in R50 means that one May farrowed, 40 pound feeder pig is supplied to the transfer row (R50) for each May farrowed, 40 pound feeder pig purchased.

Coefficients within areas V, VI, and VII are developed in the same manner and are structured very similar to area IV. The same types of activities are represented in areas V, VI, and VII as in area IV. The only difference in the activities within each area is the time period in which they take place. The same labor constraints are used for activities within areas V, VI, and VII as in area IV. The same types of farrowing and finishing constraints constrain activities in areas V, VI, and VII as in area IV. Also, as was alluded to earlier, certain transfer rows within areas V, VI, and VII are very similar to the transfer rows with non-zero coefficients in area IV. (In other words, the non-zero coefficients in subsections M1, N2, O3, and P4 are very similar.)

B. The Optimal Solution

Once the linear program is set up, it must be solved. One process that can be used in finding optimal feasible solution was discussed in section III.D.2. The optimal feasible solution of Model I is shown in tables 4.11a through 4.11c.

C. Sensitivity Analysis

Once an optimal feasible solution of the linear program is found, economic values for traits can be found from equation 3.51. The revised computable form, if you remember, can only be used, though, if the optimal mix of activities of the linear program does not change with the change of coefficients in the linear program due to the change in the h-th trait.

The process of deriving economic values will be demonstrated with three traits: backfat, feed efficiency, and average daily gain. Yet, the question still remains, "Which linear program coefficients

Table 4.11a. Optimal mix of real activities and their shadow prices: Model I

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Purchase gilts to farrow in May	A01	--	-38.3300
Purchase gilts to farrow in August	A02	--	-61.6900
Purchase gilts to farrow in November	A03	--	-83.7600
Purchase gilts to farrow in February	A04	--	-135.4000
Purchase boar to service females	A05	2 boars	--
Prepare purchased gilts for breeding and farrowing in May	A06	--	--
Prepare purchased gilts for breeding and farrowing in August	A07	--	--
Prepare purchased gilts for breeding and farrowing in November	A08	--	--
Prepare purchased gilts for breeding and farrowing in February	A09	--	--
Feed boars	A10	2 boars	--
Raise gilts to farrow in May	A11	26.3158 gilts	--
Raise gilts to farrow in August	A12	26.3158 gilts	--
Raise gilts to farrow in November	A13	1.8158 gilts	--
Raise gilts to farrow in February	A14	1.8158 gilts	--
Prepare breeding herd for breeding and farrowing in May	A15	26.3158 gilts	--

Table 4.11a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Prepare breeding herd for breeding and farrowing in August	A16	26.3158 gilts	--
Prepare breeding herd for breeding and farrowing in November	A17	26.3158 sows	--
Prepare breeding herd for breeding and farrowing in February	A18	26.3158 sows	--
Farrowing in May	A19	25.0 gilts	--
Farrowing in August	A20	25.0 gilts	--
Farrowing in November	A21	25.0 sows	--
Farrowing in February	A22	25.0 sows	--
Feed weaned May pigs to 40 pounds	A23	180.0 pigs	--
Feed weaned August pigs to 40 pounds	A24	180.0 pigs	--
Feed weaned November pigs to 40 pounds	A25	197.5 pigs	--
Feed weaned February pigs to 40 pounds	A26	197.5 pigs	--
Feed 40 pound pigs farrowed in May to 180 pounds	A27	--	-9.9110
Feed 40 pound pigs farrowed in May to 200 pounds	A28	--	-5.5934
Feed 40 pound pigs farrowed in May to 220 pounds	A29	270.8333 hogs	--
Feed 40 pound pigs farrowed in May to 240 pounds	A30	--	-1.4160
Feed 40 pound pigs farrowed in May to 260 pounds	A31	--	--
Feed 40 pound pigs farrowed in August to 180 pounds	A32	--	-9.2929

Table 4.11a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty $(z_j - c_j)$
Feed 40 pound pigs farrowed in August to 200 pounds	A33	--	-5.1227
Feed 40 pound pigs farrowed in August to 220 pounds	A34	--	-3.2287
Feed 40 pound pigs farrowed in August to 240 pounds	A35	--	-1.7405
Feed 40 pound pigs farrowed in August to 260 pounds	A36	270.8333 hogs	--
Feed 40 pound pigs farrowed in November to 180 pounds	A37	--	-.5560
Feed 40 pound pigs farrowed in November to 200 pounds	A38	195.5250 hogs	--
Feed 40 pound pigs farrowed in November to 220 pounds	A39	--	-1.2946
Feed 40 pound pigs farrowed in November to 240 pounds	A40	--	-4.1038
Feed 40 pound pigs farrowed in November to 260 pounds	A41	--	-8.1654
Feed 40 pound pigs farrowed in February to 180 pounds	A42	--	-7.7143
Feed 40 pound pigs farrowed in February to 200 pounds	A43	--	-7.6924
Feed 40 pound pigs farrowed in February to 220 pounds	A44	--	-7.8657
Feed 40 pound pigs farrowed in February to 240 pounds	A45	--	-6.3934
Feed 40 pound pigs farrowed in February to 260 pounds	A46	232.14286 hogs	--
Market May farrowed 180 pound market hogs	A47	--	--
Market May farrowed 200 pound market hogs	A48	--	--
Market May farrowed 220 pound market hogs	A49	589.8750 cwt.	--

Table 4.11a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Market May farrowed 240 pound market hogs	A50	--	--
Market May farrowed 260 pound market hogs	A51	--	-.2046
Market August farrowed 180 pound market hogs	A52	--	--
Market August farrowed 200 pound market hogs	A53	--	--
Market August farrowed 220 pound market hogs	A54	--	--
Market August farrowed 240 pound market hogs	A55	--	--
Market August farrowed 260 pound market hogs	A56	697.1250 cwt.	--
Market November farrowed 180 pound market hogs	A57	--	--
Market November farrowed 200 pound market hogs	A58	387.1400 cwt.	--
Market November farrowed 220 pound market hogs	A59	--	--
Market November farrowed 240 pound market hogs	A60	--	--
Market November farrowed 260 pound market hogs	A61	--	--
Market February farrowed 180 pound market hogs	A62	--	--
Market February farrowed 200 pound market hogs	A63	--	--
Market February farrowed 220 pound market hogs	A64	--	--
Market February farrowed 240 pound market hogs	A65	--	--
Market February farrowed 260 pound market hogs	A66	597.5357 cwt.	--

Table 4.11a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Market gilts that did not conceive in January	A67	3.2895 cwt.	--
Market gilts that did not conceive in April	A68	3.2895 cwt.	--
Market gilts that did not conceive in July	A69	3.2895 cwt.	--
Market gilts that did not conceive in October	A70	3.2895 cwt.	--
Market sows after November farrowing	A71	100.00 cwt.	--
Market sows after February farrowing	A72	100.00 cwt.	--
Market boar in November 1973	A73	8.00 cwt.	--
Market gilts culled after first farrowing (May)	A74	.825 cwt.	--
Market gilts culled after first farrowing (August)	A75	.825 cwt.	--
Purchase 40 pound feeder pigs in June	A76	92.6333 pigs	--
Purchase 40 pound feeder pigs in September	A77	92.6333 pigs	--
Purchase 40 pound feeder pigs in December	A78	--	-6.1392
Purchase 40 pound feeder pigs in March	A79	36.6179 pigs	--

Table 4.11b. Income over variable costs, Z_o: Model I

	Amount
Income	\$23,204.24

should be changed so as to represent a change in the h-th trait and by what amounts do these linear coefficients change?" This question will be discussed in detail.

1. Backfat

The trait, backfat, is one measure of the leanness of swine. Backfat measurements are taken in three places on the swine: opposite the first rib, opposite the last rib, and opposite the last lumbar vertebrae. The three measurements are averaged so as to give the swine its phenotypic backfat measurement. Backfat thickness is one criteria in grading swine carcasses when marketing swine under a grade and weight basis.

a. Linear program coefficients that will change Generally, swine with lesser amounts of fat earn a premium when marketed. This is the case when marketing swine on a grade and weight basis. Swine with less backfat, and all else equal, will receive a higher grade and thereby earn a premium. Swine, though, that are marketed strictly "by the pound" or "by the head" are generally given a straight market price for that particular day on which the swine are marketed and have no consideration for carcass grade, yield, or weight included in the price received.

Table 4.11c. Fixed input use and each fixed input's shadow price: Model I

Fixed input	Row name	Row (constraint) number i	Amount available a_{io}	Amount used $(a_{io} - x_{n+i})$	Marginal value product $(z_{n+i} - c_{n+i})$
November 1972 labor	M01	1	160	12.282	--
December 1972 labor	M02	2	196	11.819	--
January 1973 labor	M03	3	216	20.404	--
February 1973 labor	M04	4	192	27.069	--
March 1973 labor	M05	5	198	24.806	--
April 1973 labor	M06	6	160	69.904	--
May 1973 labor	M07	7	160	92.521	--
June 1973 labor	M08	8	160	96.852	--
July 1973 labor	M09	9	216	123.821	--
August 1973 labor	M10	10	208	130.437	--
September 1973 labor	M11	11	168	133.351	--
October 1973 labor	M12	12	160	150.586	--
November 1973 labor	M13	13	160	133.157	--
December 1973 labor	M14	14	196	128.225	--
January 1974 labor	M15	15	216	127.730	--
February 1974 labor	M16	16	192	127.664	--
March 1974 labor	M17	17	198	103.141	--
April 1974 labor	M18	18	160	60.057	--
May 1974 labor	M19	19	160	32.500	--
June 1974 labor	M20	20	160	32.500	--
July 1974 labor	M21	21	216	32.500	--
August 1974 labor	M22	22	208	18.249	--
May 1973 farrowing cap.	F01	23	25	25	83.4286
August 1973 farrowing cap.	F02	24	25	25	97.9304
November 1973 farrowing cap.	F03	25	25	25	109.7820
February 1974 farrowing cap.	F04	26	25	25	117.9582
Building #1 finishing cap.	R01	27	3250	3250	1.5803
Building #2 finishing cap.	R02	28	3250	3250	1.3899
Building #1 finishing cap.	R03	29	3250	2248.538	--
Building #2 finishing cap.	R04	30	3250	3250	1.1892
Boar equality	R25	35	2	2	-270.55

The swine marketed by the swine farm represented in the linear program are of two types. One type of swine that are marketed are swine fed specifically for market. The other type of swine that are marketed are swine from the breeding herd. The swine fed specifically for market are marketed on a grade and weight basis and thereby receive a premium for less backfat. The swine from the breeding herd are marketed "by the pound" and thereby receive no premium for less backfat.

Table 4.1 shows backfat thicknesses assumed for each weight group of hogs fed for market. The prices assumed for each weight group and farrowing of these same hogs are shown in table 4.5b and correspond to the backfat thicknesses shown in table 4.1. Also shown in table 4.5b are the prices received in marketing swine from the breeding herd. It can be seen that when backfat in swine fed for market changes, prices received for these market hogs change. Prices received for swine from the breeding herd do not change with backfat changes due to marketing "by the pound," though. Thus, from equation 3.3, it is known that the c_j coefficient will change for those activities in which swine fed for market are marketed.

Since backfat is a characteristic of output and not related to inputs of production, the c_j coefficients of activities in which swine fed for market are marketed are the only linear program coefficients that will change as a result of changing the backfat trait. This case was discussed in section III.E.1 as Case IA.

<u>b. Changing the appropriate c_j coefficients</u> After determining the appropriate linear program coefficients that must be changed in order to reflect a change in the h-th trait, the change of the

coefficients can be found. But before the change of the coefficients can be found, the change of the trait must be determined. As was indicated earlier, the change in the trait must not be so large as to nullify the use of the revised computable form.

The change in the backfat trait will be .15 inches. The .15 change[1] is approximately one standard deviation. This change will hopefully not change the optimal mix of activities so that the revised computable form can be used.

In order to determine the change in each c_j coefficient, the premiums must be given for different backfat thicknesses. These premiums are given in table 4.12.

Table 4.12. Premiums for backfat thicknesses for different market weights (premiums per carcass cwt.)

Market weight		Backfat thickness (inches)				
Weight	Standard yield	Under 1.2	1.2-1.3	1.3-1.6	1.6-1.9	Over 1.9
180	71.7%	$3.00	1.50	.75	0	-1.60
200	72.0	3.00	1.50	.75	0	-1.60
220	72.0	3.00	1.50	.75	0	-1.60
240	72.3	3.00	1.50	.75	0	-1.60
260	72.3	3.00	1.50	.75	0	-1.60

Table 4.12 shows that the premiums given for backfat are stepwise. This can be seen in that backfat changes from 1.3 to 1.2 or 1.6 to 1.5 indicate that no premiums are realized. A linear function can be justified to predict premiums per carcass hundredweight, though. Table 4.12 is structured for individual swine. Upon aggregation of all market swine, the function predicting premiums becomes more

[1]The .15 change in backfat will be assumed to be a .15 inch change in backfat in each hog fed for market in the remaining thesis unless stated otherwise.

nearly linear. The linear function used to predict backfat premiums per carcass hundredweight is shown in equation 4.4.

$$4.4 \qquad P = 7.34 - 4.35\ BF \qquad\qquad R^2 = 0.92$$

where P is the premium per carcass hundredweight

BF is the thickness of backfat in inches

Equation 4.4 was estimated from information given in table 4.12.

By altering equation 4.4 slightly, the premium per carcass live hundredweight for backfat can be seen. This is shown as equation 4.5.

$$4.5 \qquad P' = [7.34 - 4.35\ BF]\ [\text{Std yield}]$$

where P′ is the premium per live hundredweight

BF is the thickness of backfat

Std yield is the average percent of carcass yielded by a market hog of a designated market weight. (These can be seen in table 4.12.)

Using equation 4.5, a table of premiums can be generated for changes of +.15 and -.15 in the backfat thickness assumptions shown in table 4.1. This is shown in table 4.13.

Table 4.13. Premiums due to backfat thickness

Weight	Standard yield	Assumed backfat thickness	Backfat with +.15 change	Premium live cwt.	Backfat with -.15 change	Premium live cwt.
180	.717	1.3	1.45	.74	1.15	1.68
200	.720	1.38	1.53	.49	1.23	1.43
220	.720	1.46	1.61	.24	1.31	1.18
240	.723	1.54	1.69	-.01	1.39	.93
260	.723	1.62	1.77	-.26	1.47	.69

As in computing the c_j coefficients of the basic linear program, the premiums due to the backfat change must be discounted. Discounting the premiums is done in the same manner as discounting the c_j coefficients shown in equation 4.3. Table 4.14 shows the discounted premiums for each of the activities that are affected by a change in the trait backfat.

It was earlier indicated that prices assumed for each weight group and farrowing of swine fed for market, shown in table 4.5b, correspond to the backfat thickness shown in table 4.1. Thus, the prices used in deriving c_j coefficients for activities in which these same market hogs are marketed include certain premiums. In order to find the true change in the appropriate c_j coefficients due to a change in backfat, it is necessary to find the discounted premium for the backfat thickness, after assuming a change, less the premium for the initial backfat thickness. This process and the resulting change in c_j coefficients can be seen in table 4.15.

c. The revised computable form From section III.F, equation 3.51, the revised computable form is written

$$3.51 \qquad E.V. = \left[\frac{1}{\sum_{j^*} x_{j^*}}\right] \left[-\sum_{i,j}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j}^{n} x_{jo}\, dc_j/dt_h\right]$$

so as to find the economic value of the h-th trait. From the optimal solution of the linear program, $\sum_{j^*} x_{j^*}$, $(z_{n+i} - c_{n+i})$, and x_{jo} are given. Table 4.15 reports values of dc_j/dt_h. Now, since the c_j

Table 4.14. Discounted premiums for activities affected by changes in backfat: Model I

				Change of +.15 in backfat		Change of -.15 in backfat	
Activity	i	n	$(\frac{1}{1+.01})^n$	Premium / live cwt.	Discounted premium / live cwt.	Premium / live cwt.	Discounted premium / live cwt.
A47	.01	11	0.895336	.74	.66	1.68	1.50
A48	.01	11	0.895336	.49	.44	1.43	1.28
A49	.01	11	0.895336	.24	.21	1.18	1.06
A50	.01	12	0.886382	-.01	-.01	.93	.82
A51	.01	12	0.886382	-.26	-.23	.69	.61
A52	.01	14	0.868742	.74	.64	1.68	1.46
A53	.01	14	0.868742	.49	.43	1.43	1.24
A54	.01	14	0.868742	.24	.21	1.18	1.03
A55	.01	15	0.860054	-.01	-.01	.93	.80
A56	.01	15	0.860054	-.26	-.22	.69	.59
A57	.01	17	0.842938	.74	.62	1.68	1.42
A58	.01	17	0.842938	.49	.41	1.43	1.21
A59	.01	17	0.842938	.24	.20	1.18	.99
A60	.01	18	0.834508	-.01	-.01	.93	.78
A61	.01	18	0.834508	-.26	-.22	.69	.58
A62	.01	20	0.817900	.74	.61	1.68	1.37
A63	.01	20	0.817900	.49	.40	1.43	1.17
A64	.01	20	0.817900	.24	.20	1.18	.97
A65	.01	21	0.80972	-.01	-.01	.93	.75
A66	.01	21	0.809721	-.26	-.21	.69	.56

Table 4.15. Changes in coefficients resulting from changes in backfat: Model I

Activity[a] A_j	Initial discounted premium per live cwt.	Change of +.15 in backfat: Discounted premium live cwt.	Change of +.15 in backfat: dc_j/dt_h	Change of -.15 in backfat: Discounted premium live cwt.	Change of -.15 in backfat: dc_j/dt_h
A47	1.07	.66	-.41	1.50	+.43
A48	.86	.44	-.42	1.28	+.42
A49	.64	.21	-.43	1.06	+.42
A50	.41	-.01	-.42	.82	+.41
A51	.19	-.23	-.42	.61	+.42
A52	1.04	.64	-.40	1.46	+.42
A53	.83	.43	-.40	1.24	+.41
A54	.62	.21	-.41	1.03	+.41
A55	.40	-.01	-.41	.80	+.40
A56	.18	-.22	-.40	.59	+.41
A57	1.01	.62	-.39	1.42	+.41
A58	.81	.41	-.40	1.21	+.40
A59	.60	.20	-.40	.99	+.39
A60	.38	-.01	-.39	.78	+.40
A61	.18	-.22	-.40	.58	+.40
A62	.98	.61	-.37	1.37	+.39
A63	.79	.40	-.39	1.17	+.38
A64	.58	.20	-.38	.97	+.39
A65	.37	-.01	-.38	.75	+.38
A66	.17	-.21	-.38	.56	+.39

[a]These are the activities in which swine fed for market are marketed. Activities in which swine from the breeding herd are marketed are not included since no actual change in price is realized with a change in backfat.

coefficient of the linear program are the only coefficient affected by a change in backfat, da_{ij}/dt_h is zero.

Given $da_{ij}/dt_h = 0$ for all i and j, equation 3.51 is written as

$$3.51(a) \qquad E.V. = \left[\frac{1}{\sum\limits_{j*} x_{j*}}\right] \left[\sum_j x_{jo}\ dc_j/dt_h\right]$$

so as to find the economic value of backfat.

Table 4.16 provides the information needed to compute the economic value of backfat.

Substituting relevant information from table 4.16 into equation 3.51(a), the following economic values for the backfat are found:

Table 4.16. Elements in equation 3.51(a) needed to find the economic values of backfat for +0.15 and -0.15 changes in backfat: Model I

			+0.15 change in backfat		-0.15 change in backfat	
j	x_{jo}[a]	x_{j*}[b]	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
47	0	0	-.41	0	+.43	0
48	0	0	-.42	0	+.42	0
49	589.875	268.125	-.43	-253.6462	+.42	+247.7475
50	0	0	-.42	0	+.41	0
51	0	0	-.42	0	+.42	0
52	0	0	-.40	0	+.42	0
53	0	0	-.40	0	+.41	0
54	0	0	-.41	0	+.41	0
55	0	0	-.41	0	+.40	0
56	697.125	268.125	-.40	-278.8500	+.41	+285.8212
57	0	0	-.39	0	+.41	0
58	387.1395	193.5697	-.40	-154.8558	+.40	+154.8558
59	0	0	-.40	0	+.39	0
60	0	0	-.39	0	+.40	0
61	0	0	-.40	0	+.40	0
62	0	0	-.37	0	+.39	0
63	0	0	-.39	0	+.38	0
64	0	0	-.38	0	+.39	0
65	0	0	-.38	0	+.38	0
66	597.536	229.8214	-.38	-227.0635	+.39	+233.0389
Σ		959.6411		-914.4155		+921.4634

[a]From table 4.11a, Optimal mix of real activities and their shadow prices: Model I.

[b]Calculated by multiplying x_{jo} by the reciprocal of the average weight per head of livestock marketed as alluded to in section III.F.

For a +0.15 change in backfat

3.51(a) $$E.V. = \left[\frac{1}{\sum_{j^*} x_{j^*}}\right] \left[\sum_{j} x_{jo}\ dc_j/dt_h\right]$$

$$= \left[\frac{1}{959.6411}\right] \left[-914.4155\right]$$

$$= \$-.95$$

For a -0.15 change in backfat

3.51(a) $$E.V. = \left[\frac{1}{\sum_{j^*} x_{j^*}}\right] \left[\sum_{j} x_{jo}\ dc_j/dt_h\right]$$

$$= \left[\frac{1}{959.6411}\right] \left[+921.4634\right]$$

$$= \$.96$$

(The difference may be attributed to rounding errors involved when discounting the increment changes of the c_j coefficients.)

2. Feed efficiency

The trait, feed efficiency, in its most simple definition, is defined as the pounds of feed required to cause an animal to gain one pound. Expanding this definition, feed efficiency may be defined as the total amount of feed consumed divided by the total gain of an animal. This is shown as

4.6 $$FE = \frac{\text{Feed}}{\text{Gain}}$$

where FE is feed efficiency

Feed is pounds of feed consumed by an animal

Gain is pounds of gain by an animal because of the consumption of the feed -- see ration in table 4.3d.

a. Linear program coefficients that will change Observing equation 4.6, it can be seen that when feed efficiency of swine changes, it is a result of a change in consumption of feed and/or the amount of gain resulting from the feed consumption. In Model I, though, gain is fixed by definition in each feeding activity. Thus, in deriving an economic value for feed efficiency, the change in feed efficiency will result from a change in feed consumption of the swine.

As with the procedure in finding economic values for backfat in swine, the swine marketed are of two types when considering feed efficiency. Feed efficiency is actually characteristic of swine that are fed for market and cannot actually be considered in swine that are part of the swine breeding herd. It could be that what would be termed "feed efficiency of the swine breeding herd" may be more closely related to some other traits of breeding swine. As a result, only swine fed for market are considered in finding the economic value of feed efficiency.

Table 4.1 shows the feed efficiency assumed for each weight group of market hogs fed for market. Table 4.3d shows the basic finishing ration fed. Through the use of equation 4.1, the feed efficiency assumptions were used to find certain production coefficients of variable inputs. This was done because all feed inputs were assumed variable inputs. Using equation 3.3, certain net returns were found using the production coefficients of variable inputs derived from the feed efficiency assumptions. It can now be seen that when feed efficiency is changed, the c_j coefficient will change for each activity in which feed efficiency was used to derive net revenue coefficients and none of the other c_j coefficients will change. None of the a_{ij} vary because feed efficiency is used to derive only net return coefficients in the linear program. This case was discussed in section III.E.1 as Case IB.

b. Changing the appropriate c_j coefficients As with the trait, backfat, prior to finding the change of the coefficients, the change of the trait must be determined. Here again, the change in the trait must not be so large as to nullify the use of the revised computable form.

For the purpose of demonstrating the derivation of the economic value of feed efficiency and thus use of the revised computable form, the change in the feed efficiency trait will be .15 $\frac{\text{lbs. feed}}{\text{lbs. gain}}$. The .15 change[1] is approximately one standard deviation.

In order to determine the change in each c_j coefficient, changes in the production coefficients of variable feed inputs must be determined. Changes for production coefficients of variable feed inputs for activities involving feeding 40 pound pigs to 180 pounds are shown in table 4.17. All changes for production coefficients of variable feed inputs are derived in the same manner as the initial production coefficients of variable feed inputs, but using the change in feed efficiency instead of feed efficiency. See equation 4.1.

[1]The .15 change in feed efficiency will be assumed to be a .15 $\frac{\text{lbs. feed}}{\text{lbs. gain}}$ change in feed efficiency in each hog fed for market in the remaining thesis unless stated otherwise.

Table 4.17. Changes in production coefficients of variable feed inputs for activities involving feeding 40 pound pigs to 180 pounds: Model I

Feed input	Change of +.15 in FE			Change of -.15 in FE		
	TG	$\frac{VFI}{2000}$	Change in q_{kj}	TG	$\frac{VFI}{2000}$	Change in q_{kj}
Corn	140	$\frac{29.182 \text{ bu.}}{2000 \text{ lbs.}}$	+0.3064 bu.	140	$\frac{29.182 \text{ bu.}}{2000 \text{ lbs.}}$	-0.3064 bu.
Soybean oilmeal	140	$\frac{305.81 \text{ lbs.}}{2000 \text{ lbs.}}$	+3.2110 lbs.	140	$\frac{305.81 \text{ lbs.}}{2000 \text{ lbs.}}$	-3.2110 lbs.
Limestone	140	$\frac{15 \text{ lbs.}}{2000 \text{ lbs.}}$	+0.1575 lbs.	140	$\frac{15 \text{ lbs.}}{2000 \text{ lbs.}}$	-0.1575 lbs.
Dicalcium phosphate	140	$\frac{23 \text{ lbs.}}{2000 \text{ lbs.}}$	+0.2415 lbs.	140	$\frac{23 \text{ lbs.}}{2000 \text{ lbs.}}$	-0.2415 lbs.
Salt	140	$\frac{10 \text{ lbs.}}{2000 \text{ lbs.}}$	+0.1050 lbs.	140	$\frac{10 \text{ lbs.}}{2000 \text{ lbs.}}$	-0.1050 lbs.
Trace mineral premix	140	$\frac{2 \text{ lbs.}}{2000 \text{ lbs.}}$	+0.0210 lbs.	140	$\frac{2 \text{ lbs.}}{2000 \text{ lbs.}}$	-0.0210 lbs.
Vitamin premix	140	$\frac{10 \text{ lbs.}}{2000 \text{ lbs.}}$	+0.1050 lbs.	140	$\frac{10 \text{ lbs.}}{2000 \text{ lbs.}}$	-0.1050 lbs.
Tylosin	140	$\frac{20 \text{ gm}}{2000 \text{ lbs.}}$	+0.2100 gm	140	$\frac{20 \text{ gm}}{2000 \text{ lbs.}}$	-0.2100 lbs.

In comparing table 4.17 with table 4.9c (which illustrates the process of finding c_j coefficients), it can be seen that table 4.9c includes all variable inputs in finding c_j coefficients where table 4.17 includes only variable feed inputs. This is because in assuming a change in feed efficiency, only production coefficients of variable feed inputs change and none of the other production coefficients of variable inputs change.

Once the changes in production coefficients of the variable feed inputs have been found, the change of c_j for each of the activities affected by a change in the feed efficiency can be found. The change in net returns for activities involving feeding 40 pound pigs to 180 pounds is shown in table 4.18. Table 4.18 represents the process by which changes are found in net returns for every activity involving feeding 40 pound pigs to market weight.

The changes of the c_j coefficients due to the change in feed efficiency must be discounted to present value. Discounting the changes of appropriate c_j coefficients is done in the same manner as shown earlier in table 4.10. Table 4.19 shows the discounted changes of appropriate c_j coefficients for activities involving feeding 40 pound pigs to 180 pounds. Each change in a c_j, determined as in table 4.18, is divided equally among the three months required to raise a 40 pound pig to 180 pounds.

<u>c. The revised computable form</u> Table 4.19 shows relevant dc_j/dt_h where dt_h is the .15 change in feed efficiency. Again $\sum_{j*} x_{j*}$, $(z_{n+i} - c_{n+i})$, and x_{jo} can be found from the optimal solution, and since the c_j coefficient of the linear program is the only coefficient

Table 4.18. Change in the c_j coefficient of activity A_{27}: feeding 40 pound pigs farrowed in May to 180 pounds: Model I

Feed input	Change of +.15 in FE				Change of -.15 in FE			
	Change of q_{kj}	r_k	V_{kj}	Change of c_j	Change of q_{kj}	r_k	V_{kj}	Change of c_j
Corn	+0.3064 bu.	$2.20	$ +.67		-0.3064 bu.	$2.20	$ -.67	
Soybean oilmeal	+3.2110 lbs.	.12	+.39		-3.2110 lbs.	.12	-.39	
Limestone	+0.1575 lbs.	.02	+.01		-0.1575 lbs.	.02	-.01	
Dicalcium phosphate	+0.2415 lbs.	.10	+.02		-0.2415 lbs.	.10	-.02	
Salt	+0.1050 lbs.	.025	+.00		-0.1050 lbs.	.025	-.00	
Trace mineral premix	+0.0210 lbs.	.10	+.00		-0.0210 lbs.	.10	-.00	
Vitamin premix	+0.1050 lbs.	.60	+.06		-0.1050 lbs.	.60	-.06	
Tylosin	+0.2100 gm	.12	+.03		-0.2100 gm	.12	-.03	
Total			$+1.18	$-1.18			$-1.18	$+1.18

Table 4.19. Changes in coefficients resulting from changes in feed efficiency for activities involving feeding 40 pound pigs to 180 pounds: Model I

				Change of +.15 in FE				Change of -.15 in FE			
				Change of c_j		Discounted change of c_j		Change of c_j		Discounted change of c_j	
Activity	i	n	$(\frac{1}{1+.01})^n$	Monthly	Cumulated[a]	Monthly	Cumulated[a]	Monthly	Cumulated[a]	Monthly	Cumulated[a]
	.01	8	.9227	$-.39		$-.36		$+.39		$+.36	
A27	.01	9	.9135	-.39		-.36		+.39		+.36	
	.01	10	.9044	-.40	$-1.18	-.36	$-1.08	+.40	$+1.18	+.36	$+1.08
	.01	11	.8953	-.39		-.35		+.39		+.35	
A32	.01	12	.8864	-.39		-.35		+.39		+.35	
	.01	13	.8775	-.40	-1.18	-.35	-1.05	+.40	+1.18	+.34	+1.05
	.01	14	.8687	-.39		-.34		+.39		+.34	
A37	.01	15	.8601	-.39		-.34		+.39		+.34	
	.01	16	.8515	-.40	-1.18	-.34	-1.02	+.40	+1.18	+.34	+1.02
	.01	17	.8429	-.39		-.33		+.39		+.33	
A42	.01	18	.8345	-.39		-.33		+.39		+.33	
	.01	19	.8262	-.40	-1.18	-.33	-.99	+.40	+1.18	+.33	+.99

[a]Activities A27, A32, A37, and A42 (plus other activities involving feeding 40 pound pigs to different market weights) last for periods of three months or more and therefore must have each month's net return discounted to present value separately. After discounting each month's net revenue to present value, the related month's net return may be added together to find the change in c_j for the activity.

affected by a change in feed efficiency, as with backfat, da_{ij}/dt_h is zero. Therefore, all the unknowns of equation 3.51 are known so that the economic value of the feed efficiency trait can be found.

Since $da_{ij}/dt_h = 0$ for all i and j, equation 3.51(a) can be used to find the economic value of feed efficiency. Given table 4.20 where all $\sum_{j*} x_{j*}$, x_{jo}, and dc_j/dt_h are listed and where j* identifies an activity that produces an animal that has a change in feed efficiency, the economic value of feed efficiency can easily be found.

Substituting elements from table 4.20 into equation 3.51(a), the following economic values for the trait, feed efficiency, are found:

For a +0.15 change in feed efficiency

3.51(a) $$E.V. = [\frac{1}{969.334}] [-\$1394.488] = \$-1.44$$

For a -0.15 change in feed efficiency

3.51(a) $$E.V. = [\frac{1}{969.334}] [+1399.517] = \$1.44$$

In comparing table 4.20 to table 4.16, it can be seen that different activities are included in the tables. This is due to the fact that changes in backfat affect marketing activities of swine fed for market and changes in feed efficiency affect feeding activities of swine fed for market. Table 4.20 shows x_{jo} equal to x_{j*} for each activity since the levels of the activities are in numbers of livestock. This is consistent with the explanation given in section III.F.

Table 4.20. Elements in equation 3.51(a) needed to find the economic values of feed efficiency for +0.15 and -0.15 changes in feed efficiency: Model I

j^a	x_{jo}	x_{j*}	+0.15 change in FE dc_j/dt_h	+0.15 change in FE $x_{jo}\ dc_j/dt_h$	-0.15 change in FE dc_j/dt_h	-0.15 change in FE $x_{jo}\ dc_j/dt_h$
A27	0	0	$-1.08	0	$+1.08	0
A28	0	0	-1.22	0	+1.24	0
A29	270.83333	270.833	-1.37	$-371.042	+1.38	$+373.750
A30	0	0	-1.53	0	+1.53	0
A31	0	0	-1.68	0	+1.68	0
A32	0	0	-1.05	0	+1.05	0
A33	0	0	-1.19	0	+1.19	0
A34	0	0	-1.34	0	+1.34	0
A35	0	0	-1.48	0	+1.49	0
A36	270.83333	270.833	-1.63	-441.458	+1.63	+441.458
A37	0	0	-1.02	0	+1.02	0
A38	195.525	195.525	-1.16	-226.809	+1.16	+226.809
A39	0	0	-1.30	0	+1.30	0
A40	0	0	-1.44	0	+1.44	0
A41	0	0	-1.58	0	+1.58	0
A42	0	0	-0.99	0	+ .99	0
A43	0	0	-1.12	0	+1.12	0
A44	0	0	-1.26	0	+1.26	0
A45	0	0	-1.40	0	+1.40	0
A46	232.14286	232.143	-1.53	-355.179	+1.54	+357.500
Σ		969.334		-1394.488		+1399.517

[a]These are the activities in which swine fed for market are fed. Activities in which swine of the breeding herd are fed are not included since it is assumed that the feed efficiency of the swine in the breeding herd is more closely related to some other trait.

3. Average daily gain

The trait, average daily gain, in its most simple definition, is defined as the pounds of gain by an animal per day. Average daily gain may also be defined as the total pounds of gain by an animal divided by the number of days it takes an animal to make the total gain. This is shown as

$$4.7 \qquad ADG = \frac{Gain}{Days}$$

where ADG is average daily gain

Gain is pounds of gain by an animal because of consumption of feed -- see ration in table 4.3d

Days is the number of days it takes an animal to make the total gain

a. Linear program coefficients that will change   Equation 4.7 shows that when average daily gain of swine changes, it is a result of a change in the amount of gain by the swine and/or the number of days it takes the swine to make the total gain. As with feed efficiency, in deriving the economic value of average daily gain, the amount of gain of the swine must not change. This, again, is because gain is fixed by definition in each feeding activity, but also for another reason; because of the relationship of feed efficiency and average daily gain through total gain.

If a change in average daily gain was reflected through a change in gain, both average daily gain and feed efficiency would change. Both average daily gain and feed efficiency cannot change simultaneously since the economic value of a trait must be the change in profit as a direct result of changing the one specific trait and must not include any profit change due to a correlated trait. Thus, in deriving an economic value for average daily gain, the change in average daily gain will result from a change in the number of days it takes to make the gain and not because of a change in total gain.

As with feed efficiency, average daily gain is characteristic of swine that are fed for market and is generally not considered in swine that are part of the swine breeding herd. Swine that are part of the breeding herd are not fed so that they may gain. They are fed enough feed for body maintenance. As a result, only swine fed for market are considered in finding the economic value of average daily gain.

Table 4.1 shows the average daily gain assumed for each weight group of market hogs fed for market. These average daily gain assumptions were used directly and indirectly in finding two types of a_{ij} coefficients and in finding c_j coefficients of certain activities. The average daily gain assumptions were used directly in determining production coefficients of fixed labor inputs. This was done by determining the length of time the swine were fed and then alloting the labor required appropriately among the months within the time period. The average daily gain assumptions were used indirectly in determining production coefficients of fixed inputs for the finishing area. This was done by determining the length of time the swine were fed and then determining the finishing area required per hog from table 4.6c by knowing the weight of the hog and the season of the year within the time period. The average daily gain assumptions were used directly in determining the power and fuel portion of the total variable cost of feeding swine. This was done by proportioning power and fuel costs for each activity based on the power and fuel cost in the time period of producing swine fed for market.

It can now be seen that with a change in average daily gain, both production coefficients of fixed inputs and net returns generated by performing certain activities may be changed. The net return coefficients actually change due to a change in the production coefficients of variable inputs. This case was discussed in section III.E.1 as Case IIIB.

b. Changing the appropriate c_j and a_{ij} coefficients   Again, the change in the trait must first be determined. For the purpose of demonstrating the derivation of the economic value of average daily gain using the revised computable form, the change in the average daily gain trait will be .15 lbs. of gain per day. The .15 change[1] is approximately one standard deviation. This change will hopefully not change the optimal mix of activities so that the revised computable form can be used.

In order to determine the change in each a_{ij} coefficient that changes, changes in the number of days swine are fed must be determined. In order to determine the change in each c_j coefficient that changes, changes in the power and fuel portion of variable cost must be determined. Changes in the number of days the swine are fed are shown in table 4.21.

Once the changes in the number of days swine are fed are found, the changes in the appropriate a_{ij} coefficients can be found. The changes in the production coefficients of fixed labor inputs are found by multiplying the change in days fed divided by 30.4166 days per month times the hours of labor required in the month in which the change takes place (which is the last month of the feeding period). These changes are shown in table 4.22. The changes in the production coefficients of fixed finishing area inputs cannot be found as

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[1]The .15 change in average daily gain will be assumed to be a .15 lbs. of gain per day change in average daily gain in each hog fed for market in the remaining thesis unless stated otherwise.

Table 4.21. Changes in number of days swine are fed reflecting a change in average daily gain

Market weight	Days fed under initial ADG: Initial ADG	Days fed under initial ADG: Days fed	+0.15 change in ADG: Days fed	+0.15 change in ADG: Change in days fed	-0.15 change in ADG: Days fed	-0.15 change in ADG: Change in days fed
180	1.5246	91.83	83.602	-8.228	101.8478	+10.018
200	1.5804	101.24	92.464	-8.776	111.857	+10.617
220	1.6298	110.44	101.135	-9.305	121.638	+11.198
240	1.6728	119.56	109.7213	-9.839	131.337	+11.777
260	1.7109	128.59	118.2223	-10.368	140.9443	+12.354

systematically as the production coefficients of fixed labor inputs. The changes in the production coefficients of fixed finishing area inputs are found by analyzing the new feeding period of each activity in which market hogs fed for market are fed, where the new feeding period results from a change in average daily gain. Upon analyzing the new feeding period, the season of the year in which the swine are fed is known. Using the season of the year in which the swine are fed and the size of the swine through the feeding period (i.e., weight of the market hog), the finishing area needed per market hog can be found using table 4.6c. Now, if the area needed per market hog differs from the one indicated under the original average daily gain assumption (shown in table 4.6d), the difference between the two coefficients is the change in the production coefficient of fixed finishing area inputs. If the same a_{ij} coefficient is used, there obviously is no change. The changes for production coefficients of fixed inputs for finishing area are shown in table 4.23.

Table 4.22. Changes in production coefficients of fixed labor inputs: Model I

Row[a] no. i	Activity j	+0.15 change in ADG: Change in days fed	+0.15 change in ADG: 1 ÷ Days in month	+0.15 change in ADG: Hours labor required for total month	+0.15 change in ADG: da_{ij}/dt_h	-0.15 change in ADG: Change in days fed	-0.15 change in ADG: 1 ÷ Days in month	-0.15 change in ADG: Hours labor required for total month	-0.15 change in ADG: da_{ij}/dt_h
11	A27	-8.228	$\frac{1}{30.4166}$	.15	-.04				
12	A27					+10.018	$\frac{1}{30.4166}$	.15	+.05
12	A28	-8.776	$\frac{1}{30.4166}$	.15	-.04				
12	A28					+10.617	$\frac{1}{30.4166}$	.15	+.05
12	A29	-9.305	$\frac{1}{30.4166}$	.15	-.04				
12	A29					+11.198	$\frac{1}{30.4166}$	.15	+.06
12	A30	-9.839	$\frac{1}{30.4166}$	.15	-.05				
13	A30					+11.777	$\frac{1}{30.4166}$	.15	+.06
12	A31	-10.368	$\frac{1}{30.4166}$	.15	-.01				
13	A31				-.04				
13	A31					+12.354	$\frac{1}{30.4166}$	.15	+.06
14	A32	-8.228	$\frac{1}{30.4166}$	.15	-.04				

[a] See table 4.11c for reference.

Table 4.22. Continued

Row no. i	Activity j	+0.15 change in ADG: Change in days fed	+0.15 change in ADG: 1 ÷ Days in month	+0.15 change in ADG: Hours labor required for total month	+0.15 change in ADG: da_{ij}/dt_h	-0.15 change in ADG: Change in days fed	-0.15 change in ADG: 1 ÷ Days in month	-0.15 change in ADG: Hours labor required for total month	-0.15 change in ADG: da_{ij}/dt_h
15	A32					+10.018	$\frac{1}{30.4166}$	.15	+.05
15	A33	-8.776	$\frac{1}{30.4166}$	.15	-.04				
15	A33					+10.617	$\frac{1}{30.4166}$	.15	+.05
15	A34	-9.305	$\frac{1}{30.4166}$	.15	-.04				
15	A34					+11.198	$\frac{1}{30.4166}$	.15	+.06
15	A35	-9.839	$\frac{1}{30.4166}$	.15	-.05				
16	A35					+11.777	$\frac{1}{30.4166}$	.15	+.06
15	A36	-10.368	$\frac{1}{30.4166}$	.15	-.01				
16	A36				-.04				
16	A36					+12.354	$\frac{1}{30.4166}$	.15	+.06
17	A37	-8.228	$\frac{1}{30.4166}$	.14	-.04				

Table 4.22. Continued

Row no. i	Activity j	+0.15 change in ADG: Change in days fed	+0.15 change in ADG: 1 ÷ Days in month	+0.15 change in ADG: Hours labor required for total month	+0.15 change in ADG: da_{ij}/dt_h	-0.15 change in ADG: Change in days fed	-0.15 change in ADG: 1 ÷ Days in month	-0.15 change in ADG: Hours labor required for total month	-0.15 change in ADG: da_{ij}/dt_h
18	A37					+10.018	$\frac{1}{30.4166}$	.15	+.05
18	A38	-8.776	$\frac{1}{30.4166}$	.15	-.04				
18	A38					+10.617	$\frac{1}{30.4166}$	.15	+.05
18	A39	-9.305	$\frac{1}{30.4166}$	.15	-.04				
18	A39					+11.198	$\frac{1}{30.4166}$	.15	+.05
18	A40	-9.839	$\frac{1}{30.4166}$	.15	-.05				
19	A40					+11.777	$\frac{1}{30.4166}$	.15	+.05
18	A41	-10.368	$\frac{1}{30.4166}$	.15	-.01				
19	A41				-.04				
19	A41					+12.354	$\frac{1}{30.4166}$	.15	+.06
20	A42	-8.228	$\frac{1}{30.4166}$	.14	-.04				

Table 4.22. Continued

Row no. i	Activity j	+0.15 change in ADG				-0.15 change in ADG			
		Change in days fed	1 ÷ Days in month	Hours labor required for total month	da_{ij}/dt_h	Change in days fed	1 ÷ Days in month	Hours labor required for total month	da_{ij}/dt_h
21	A42					+10.018	$\frac{1}{30.4166}$	.14	+.05
21	A43	-8.776	$\frac{1}{30.4166}$	.14	-.04				
21	A43					+10.617	$\frac{1}{30.4166}$	.14	+.05
21	A44	-9.305	$\frac{1}{30.4166}$	.14	-.04				
21	A44					+11.198	$\frac{1}{30.4166}$	.14	+.05
21	A45	-9.839	$\frac{1}{30.4166}$	.14	-.05				
22	A45					+11.777	$\frac{1}{30.4166}$	.14	+.05
21	A46	-10.368	$\frac{1}{30.4166}$	.14	-.05				
22	A46					+12.354	$\frac{1}{30.4166}$	.14	+.05

Table 4.23. Changes in production coefficients of fixed finishing area inputs: Model I

Row no. i	Activity[a] j	+0.15 change in ADG: Initial a_{ij} coefficient	+0.15 change in ADG: New a_{ij} coefficient	$\frac{da_{ij}}{dt_h}$	-0.15 change in ADG: Initial a_{ij} coefficient	-0.15 change in ADG: New a_{ij} coefficient	$\frac{da_{ij}}{d_{th}}$
27	A27	12	12	0	12	11	-1
27	A28	12	12	0	12	11.5	-0.5
27	A29	12	12	0	12	12	0
27	A30	12.5	12.5	0	12.5	12.5	0
27	A31	13	13	0	13	13	0
28	A32	11	11	0	11	10	-1
28	A33	11	11	0	11	10.5	-0.5
28	A34	11	11	0	11	11	0
28	A35	11.5	11.5	0	11.5	11.5	0
28	A36	12	12	0	12	12	0
29	A37	10	10	0	10	11	+1
29	A38	11.5	11.5	0	11.5	11.5	0
29	A39	12	12	0	12	12	0
29	A40	12.5	12.5	0	12.5	12.5	0
29	A41	13	13	0	13	13	0
30	A42	12	12	0	12	12	0
30	A43	12.5	12.5	0	12.5	12.5	0
30	A44	13	13	0	13	13	0
30	A45	13.5	13.5	0	13.5	13.5	0
30	A46	14	14	0	14	14	0

[a]These are the activities in which swine fed for market are fed. Activities in which swine of the breeding herd are fed are not included since they are fed enough feed for body maintenance and not for gain.

With the changes in the number of days swine are fed known, the changes in the appropriate c_j coefficients can be found. In order to determine the change in each c_j coefficient, though, changes in the power and fuel portion of the variable cost of the activity must be determined. Changes in the power and fuel portion of the variable cost of the appropriate activities are shown in table 4.24.

Through equation 3.3, it is known that ΔV_j is equal to $-\Delta c_j$, thus with the change in power and fuel portion of variable cost in each activity, the change in net returns generated by the activity is known. But again, the changes of c_j must be discounted to present value. The procedure of discounting the changes of appropriate c_j coefficients is done in the same manner as shown earlier in table 4.10. The discounted change of appropriate c_j coefficients due to a change in average daily gain are shown in table 4.25.

c. The revised computable form From tables 4.22, 4.23, and 4.25, changes in a_{ij} and c_j coefficients, due to the 0.15 change in average daily gain, are given. Again, $\sum_{j*} x_{j*}$, $(z_{n+i} - c_{n+i})$, and x_{jo} can be found from the optimal solution. Thus, the elements of equation 3.51 are known so that the economic value of the average daily gain trait can be found.

Since $da_{ij}/dt_h \neq 0$ for certain i and j, equation 3.51 must be used to find the economic value of the average daily gain trait. Given tables 4.26 and 4.27 where all $\sum_{j*} x_{j*}$, x_{jo}, $(z_{n+i} - c_{n+i})$, da_{ij}/dt_h, and dc_j/dt_h are listed for +0.15 and -0.15 changes in average daily gain, respectively, and each j* identifies an activity

Table 4.24. Changes in the power and fuel portion of the variable cost of appropriate activities: Model I

Activity[a]	Initial power and fuel portion of the variable cost	Initial days fed	+0.15 change in ADG: Days fed	+0.15 change in ADG: Change in days fed	+0.15 change in ADG: Change in power and fuel (ΔV_j)	-0.15 change in ADG: Days fed	-0.15 change in ADG: Change in days fed	-0.15 change in ADG: Change in power and fuel (ΔV_j)
A27	$1.00	83.602	75.374	-8.228	$-.09	93.62	+10.018	$+.11
A28	1.10	92.464	83.688	-8.776	-.10	103.081	+10.617	+.12
A29	1.20	101.135	91.830	-9.305	-.10	112.333	+11.198	+.12
A30	1.30	109.7213	99.8823	-9.839	-.11	121.4983	+11.777	+.13
A31	1.40	118.2223	107.8543	-10.368	-.11	130.577	+12.354	+.13
A32	1.00	83.602	75.374	-8.228	-.09	93.62	+10.018	+.11
A33	1.10	92.464	83.688	-8.776	-.10	103.081	+10.617	+.12
A34	1.20	101.135	91.830	-9.305	-.10	112.333	+11.198	+.12
A35	1.30	109.7213	99.8823	-9.839	-.11	121.4983	+11.777	+.13
A36	1.40	118.2223	107.8543	-10.368	-.11	130.577	+12.354	+.13
A37	1.00	83.602	75.374	-8.228	-.01	93.62	+10.018	+.11
A38	1.10	92.464	83.688	-8.776	-.10	103.081	+10.617	+.12
A39	1.20	101.135	91.830	-9.305	-.10	112.333	+11.198	+.12
A40	1.30	109.7213	99.8823	-9.839	-.11	121.4983	+11.777	+.13
A41	1.40	118.223	107.8543	-10.368	-.11	130.577	+12.354	+.13
A42	1.00	83.602	75.374	-8.228	-.01	93.62	+10.018	+.11
A43	1.10	92.464	83.688	-8.776	-.10	103.081	+10.617	+.12
A44	1.20	101.135	91.830	-9.305	-.10	112.333	+11.198	+.12
A45	1.30	109.7213	99.8823	-9.839	-.11	121.4983	+11.777	+.13
A46	1.40	118.223	107.8543	-10.368	-.11	130.577	+12.354	+.13

[a]These are the activities in which swine fed for market are fed. Activities in which swine of the breeding herd are fed are not included since they are fed enough feed for body maintenance and not for gain.

Table 4.25. Changes in c_j coefficients resulting from changes in average daily gain: Model I

				Changes of +0.15 in ADG		Changes of -0.15 in ADG	
Activity	i	n[a]	$(\frac{1}{1+0.01})^n$	Change of c_j	Discounted change of c_j	Change of c_j	Discounted change of c_j
A27	0.01	10	0.9044	$+.09	$+.08	$-.11	$-.10
A28	0.01	11	0.8953	+.10	+.09	-.12	-.11
A29	0.01	11	0.8953	+.10	+.09	-.12	-.11
A30	0.01	11	0.8953	+.11	+.10	-.13	-.12
A31	0.01	12	0.8864	+.11	+.10	-.13	-.12
A32	0.01	13	0.8775	+.09	+.08	-.11	-.10
A33	0.01	14	0.8687	+.10	+.09	-.12	-.11
A34	0.01	14	0.8687	+.10	+.09	-.12	-.11
A35	0.01	14	0.8687	+.11	+.10	-.13	-.11
A36	0.01	15	0.8601	+.11	+.10	-.13	-.11
A37	0.01	16	0.8515	+.09	+.08	-.11	-.09
A38	0.01	16	0.8515	+.10	+.09	-.12	-.10
A39	0.01	16	0.8515	+.10	+.09	-.12	-.10
A40	0.01	17	0.8429	+.11	+.09	-.13	-.11
A41	0.01	18	0.8345	+.11	+.09	-.13	-.11
A42	0.01	19	0.8262	+.09	+.08	-.11	-.09
A43	0.01	20	0.8179	+.10	+.08	-.12	-.10
A44	0.01	20	0.8179	+.10	+.08	-.12	-.10
A45	0.01	20	0.8179	+.11	+.09	-.13	-.11
A46	0.01	21	0.8097	+.11	+.09	-.13	-.11

[a]Since the end of the period of each activity is the time when the power and fuel cost change is realized, it is from that point in time that the change is discounted.

Table 4.26. Elements in equation 3.51 needed to find the economic value of average daily gain for a +0.15 change in average daily gain: Model I

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$[(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h]$	dc_j/dt_h	$[x_{jo}\ dc_j/dt_h]$
27	11	0	0	0	-0.04	0	$+.08	0
	27			1.58030	0	0		
28	12	0	0	0	-0.04	0	+.09	0
	27			1.5803	0	0		
29	12	270.8333	270.8333	0	-0.04	0	+.09	$+24.38
	27			1.5803	0	0		
30	12	0	0	0	-0.05	0	+.10	0
	27			1.5803	0	0		
31	12	0	0	0	-0.01	0	+.10	0
	13			0	-0.04	0		
	27			1.5803	0	0		
32	14	0	0	0	-0.04	0	+.08	0
	28			1.3899	0	0		
33	15	0	0	0	-0.04	0	+.09	0
	28			1.3899	0	0		
34	15	0	0	0	-0.04	0	+.09	0
	28			1.3899	0	0		
35	15	0	0	0	-0.05	0	+.10	0
	28			1.3899	0	0		
36	15	270.8333	270.8333	0	-0.01	0	+.10	+27.08
	16			0	-0.04	0		
	28			1.3899	0	0		
37	17	0	0	0	-0.04	0	+.08	0
	29			0	0	0		

Table 4.26. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$[(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h]$	dc_h/dt_h	$[x_{jo}\ dc_j/dt_h]$
38	18 29	195.525	195.525	0 0	-0.04 0	0 0	+.09	+17.60
39	18 29	0	0	0 0	-0.04 0	0 0	+.09	0
40	18 29	0	0	0 0	-0.05 0	0 0	+.09	0
41	18 19 29	0	0	0 0 0	-0.01 -0.04 0	0 0 0	+.09	0
42	20 30	0	0	0 1.1892	-0.04 0	0 0	+.08	0
43	21 30	0	0	0 1.1892	-0.04 0	0 0	+.08	0
44	21 30	0	0	0 1.1892	-0.04 0	0 0	+.08	0
45	21 30	0	0	0 1.1892	-0.05 0	0 0	+.09	0
46	20 21 30	232.1429	232.1429	0 0 1.1892	-0.01 -0.04 0	0 0 0	+.09	+20.89
Σ			969.334			0		89.95

Table 4.27. Elements in equation 3.51 needed to find the economic value of average daily gain for a -0.15 change in average daily gain: Model I

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$[(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h]$	dc_j/dt_h	$[x_{jo}\ dc_j/dt_h]$
27	12	0	0	0	+.05	0	$-.10	0
	27			1.5803	-1	0		
28	12	0	0	0	+.05	0	-.11	0
	27			1.5803	-0.5	0		
29	12	270.8333	270.8333	0	+0.06	0	-.11	$-29.79
	27			1.5803	0	0		
30	13	0	0	0	0	0	-.12	0
	27			1.5803	0	0		
31	13	0	0	0	+0.06	0	-.12	0
	27			1.5803	0	0		
32	15	0	0	0	+0.05	0	-.10	0
	28			1.3899	-1	0		
33	15	0	0	0	+0.05	0	-.11	0
	28			1.3899	-0.5	0		
34	15	0	0	0	+0.06	0	-.11	0
	28			1.3899	0	0		
35	16	0	0	0	+0.06	0	-.11	0
	28			1.3899	0	0		
36	16	270.8333	270.8333	0	+0.06	0	-.11	-29.79
	28			1.3899	0	0		
37	18	0	0	0	+0.05	0	-.09	0
	29			0	+1	0		

Table 4.27. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$[(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h]$	dc_j/dt_h	$[x_{jo}\ dc_j/dt_h]$
38	18	195.525	195.525	0	+0.05	0	-.10	-19.55
	29			0	0	0		
39	18	0	0	0	+0.05	0	-.10	0
	29			0	0	0		
40	19	0	0	0	+0.05	0	-.11	0
	29			0	0	0		
41	19	0	0	0	+0.06	0	-.11	0
	29			0	0	0		
42	21	0	0	0	+0.05	0	-.09	0
	30			1.1892	0	0		
43	21	0	0	0	+0.05	0	-.10	0
	30			1.1892	0	0		
44	21	0	0	0	+0.05	0	-.10	0
	30			1.1892	0	0		
45	22	0	0	0	+0.05	0	-.11	0
	30			1.1892	0	0		
46	22	232.1429	232.143	0	+0.05	0	-.11	-25.54
	30			1.1892	0	0		
Σ			969.334			0		-104.67

that produces an animal that has a change in average daily gain, the economic value of average daily gain can be easily found.

For a +0.15 change in average daily gain

$$3.51 \qquad E.V. = [\frac{1}{\sum_{j^*} x_{j^*}}] [-\sum_{i,j} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_j x_{jo}\, dc_j/dt_h]$$

$$= [\frac{1}{969.334}] [0 + 89.95]$$

$$= \$.09$$

Substituting elements from table 4.26 into equation 3.51, the following economic value for the trait, average daily gain, is found:

For a -0.15 change in average daily gain

$$3.51 \qquad E.V. = [\frac{1}{\sum_{j^*} x_{j^*}}] [-\sum_{i,j} (z_{n+i} - c_{n+i})\, x_{o}\, da_{ij}/dt_h + \sum_j x_{jo}\, dc_j/dt_h]$$

$$= [\frac{1}{969.334}] [0 + (-104.67)]$$

$$= \$-.11$$

4. Finding additional economic values using the revised computable form

Economic values of backfat, feed efficiency, and average daily gain reported in the previous sections have been values based on 0.15 (approximately one standard deviation) changes in the traits. Additional economic values may be found using the revised computable form based on 0.30 (approximately two standard deviation) changes in the traits.

The procedure followed in deriving the economic value of each respective trait based on 0.30 changes is identical to the procedure followed in deriving the economic values of backfat, feed efficiency, and average daily gain based on 0.15 changes. Since the procedure in deriving the economic values, using the revised computable form, is always the same, independent of the amount of change in the trait, the tables that demonstrate the derivation of changes in the linear program coefficients will not be shown in the following. Instead, a table of only changes of the relevant linear program coefficients will be shown so as to show the values to be substituted into the revised computable form to derive the economic value of the h-th trait.

a. Backfat Given table 4.28 where all $\sum_{j*} x_{j*}$, x_{jo}, and dc_j/dt_h are listed where j* identifies an activity that produces an animal that has a change in backfat, the economic value of backfat can be found.

Substituting elements from table 4.28 into equation 3.51(a), the following economic values for backfat are found:

For a +0.30 change in backfat

3.51(a) $$\text{E.V.} = \left[\frac{1}{959.6411}\right] [-1826.032] = \$-1.90$$

For a -0.30 change in backfat

3.51(a) $$\text{E.V.} = \left[\frac{1}{959.6411}\right] [1833.0804] = \$1.91$$

(The difference, again, may be attributed to rounding errors involved when discounting the increment changes of the c_j coefficients.)

Table 4.28. Elements in equation 3.51(a) needed to find the economic values of backfat for +0.30 and -0.30 changes in backfat: Model I

			+0.30 change in backfat		-0.30 change in backfat	
j	x_{jo}	x_j	dc_j/dt_h	$[x_{jo}\ dc_j/dt_h]$	dc_j/dt_h	$[x_{jo}\ dc_j/dt_h]$
47	0	0	-.83	0	+.85	0
48	0	0	-.84	0	+.84	0
49	589.875	268.125	-.85	-501.3938	+.84	495.4950
50	0	0	-.84	0	+.84	0
51	0	0	-.84	0	+.84	0
52	0	0	-.81	0	+.82	0
53	0	0	-.81	0	+.82	0
54	0	0	-.82	0	+.81	0
55	0	0	-.81	0	+.81	0
56	697.125	268.125	-.81	-564.6713	+.82	571.6425
57	0	0	-.78	0	+.79	0
58	387.1395	193.5697	-.79	-305.8402	+.79	305.8402
59	0	0	-.79	0	+.79	0
60	0	0	-.78	0	+.80	0
61	0	0	-.79	0	+.79	0
62	0	0	-.76	0	+.77	0
63	0	0	-.77	0	+.76	0
64	0	0	-.77	0	+.77	0
65	0	0	-.76	0	+.77	0
66	597.536	229.8214	-.76	-454.1274	+.77	460.1027
Σ		959.6411		-1826.0327		1833.0804

b. Feed efficiency Table 4.29 provides the elements in equation 3.51(a) so that the economic value of feed efficiency can be found.

Substituting elements from table 4.29 into equation 3.51(a), the following economic values for feed efficiency are found:

For a +0.30 change in feed efficiency

$$3.51(a) \qquad E.V. = \left[\frac{1}{969.334}\right] [-2789.7284] = \$-2.88$$

Table 4.29. Elements in equation 3.51(a) needed to find economic values of feed efficiency for +0.30 and -0.30 changes in feed efficiency: Model I

			+0.30 change in FE		-0.30 change in FE	
j	x_{jo}	x_j	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
27	0	0	-2.17	0	$+2.16	0
28	0	0	-2.46	0	+2.47	0
29	270.8333	270.833	-2.75	-744.7917	+2.76	747.4999
30	0	0	-3.06	0	+3.06	0
31	0	0	-3.36	0	+3.36	0
32	0	0	-2.09	0	+2.09	0
33	0	0	-2.38	0	+2.39	0
34	0	0	-2.67	0	+2.68	0
35	0	0	-2.97	0	+2.97	0
36	270.8333	270.833	-3.26	-882.9167	+3.26	882.9167
37	0	0	-2.03	0	+2.03	0
38	195.525	195.525	-2.31	-451.6628	+2.31	451.6628
39	0	0	-2.60	0	+2.60	0
40	0	0	-2.88	0	+2.88	0
41	0	0	-3.16	0	+3.15	0
42	0	0	-1.97	0	+1.98	0
43	0	0	-2.24	0	+2.25	0
44	0	0	-2.52	0	+2.52	0
45	0	0	-2.79	0	+2.80	0
46	232.14286	232.143	-3.06	-710.3572	+3.07	712.6786
Σ		969.334		-2789.7284		2794.7580

For a -0.30 change in feed efficiency

3.51(a) $$\text{E.V.} = \left[\frac{1}{969.334}\right]\ [2794.758] = \$2.88$$

c. Average daily gain Table 4.30 lists all $\sum_{j*} x_{j*}$, x_{jo}, $(z_{n+i} - c_{n+i})$, da_{ij}/dt_h, and dc_j/dt_h for a +.30 change in average daily gain where j* identifies an activity that produces an animal that has a change in average daily gain. Table 4.31 lists all $\sum_{j*} x_{j*}$,

Table 4.30. Elements in equation 3.51 needed to find the economic value of average daily gain for a +0.30 change in average daily gain: Model I

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
27	11 27	0	0	0 1.5803	-0.07 0	0 0	+.15	0
28	11 12 27	0	0	0 0 1.5803	-0.03 -0.04 0	0 0 0	+.16	0
29	12 27	270.8333	270.8333	0 1.5803	-0.07 0	0 0	+.17	46.042
30	12 27	0	0	0 1.5803	-0.10 0	0 0	+.18	0
31	12 13 27	0	0	0 0 1.5803	-0.06 -0.04 0	0 0 0	+.19	0
32	14 28	0	0	0 1.3899	-0.07 0	0 0	+.14	0
33	14 15 28	0	0	0 0 1.3899	-0.03 -0.04 0	0 0 0	+.16	0
34	15 28	0	0	0 1.3899	-0.08 0	0 0	+.17	0
35	15 28	0	0	0 1.3899	-0.10 0	0 0	+.18	0

Table 4.30. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
	15			0	-0.06	0		
36	16	270.8333	270.8333	0	-0.04	0	+.18	48.750
	28			1.3899	0	0		
37	17	0	0	0	-0.07	0	+.14	0
	29			0	0	0		
	17			0	-0.03	0		
38	18	195.525	195.525	0	-0.04	0	+.15	29.329
	29			0	-1	0		
39	18	0	0	0	-0.07	0	+.16	0
	29			0	0	0		
40	18	0	0	0	-0.10	0	+.17	0
	29			0	0	0		
	18			0	-0.06	0		
41	19	0	0	0	-0.04	0	+.18	0
	29			0	0	0		
42	20	0	0	0	-0.06	0	+.13	0
	30			1.1892	-1	0		
	20			0	-0.02	0		
43	21	0	0	0	-0.04	0	+.15	0
	30			1.1892	-1	0		
44	21	0	0	0	-0.07	0	+.16	0
	30			1.1892	0	0		

Table 4.30. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
45	21	0	0	0	-0.10	0	+.17	0
	30			1.1892	0	0		
	21			0	-0.06	0		
46	22	232.1429	232.1429	0	-0.04	0	+.17	39.464
	30			1.1892	0	0		
Σ			969.334			0		163.585

Table 4.31. Elements in equation 3.51 needed to find the economic value of average daily gain for a -0.30 change in average daily gain

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
27	12	0	0	0	+0.11	0	-.21	0
	27			1.5803	-1	0		
	12			0	+0.10	0		
28	13	0	0	0	+0.02	0	-.24	0
	27			1.5803	-0.5	0		
	12			0	+0.06	0		
29	13	270.8333	270.8333	0	+0.07	0	-.24	-64.999
	27			1.5803	0	0		
30	13	0	0	0	+0.12	0	-.25	0
	27			1.5803	0	0		
	13			0	+0.11	0		
31	14	0	0	0	+0.02	0	-.27	0
	27			1.5803	0	0		
32	15	0	0	0	+0.11	0	-.21	0
	28			1.3899	-1	0		
	15			0	+0.10	0		
33	16	0	0	0	+0.02	0	-.23	0
	28			1.3899	-0.5	0		
	15			0	+0.06	0		
34	16	0	0	0	+0.07	0	-.24	0
	28			1.3899	0	0		

Table 4.31. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
35	16	0	0	0	+0.12	0	-.25	0
	28			1.3899	0	0		
36	16	270.8333	270.8333	0	+0.11	0	-.26	-70.417
	17			0	+0.02	0		
	28			1.3899	0	0		
37	18	0	0	0	+0.11	0	-.21	0
	29			0	+1	0		
38	18	195.525	195.525	0	+0.10	0	-.22	-43.016
	19			0	+0.02	0		
	29			0	0	0		
39	18	0	0	0	+0.06	0	-.23	0
	19			0	+0.07	0		
	29			0	0	0		
40	19	0	0	0	+0.12	0	-.24	0
	29			0	0	0		
41	19	0	0	0	+0.11	0	-.26	0
	20			0	+0.02	0		
	29			0	0	0		
42	21	0	0	0	+0.11	0	-.20	0
	30			1.1892	0	0		
43	21	0	0	0	+0.10	0	-.21	0
	22			0	+0.02	0		
	30			1.1892	0	0		

Table 4.31. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
	21			0	+0.06	0		
44	22	0	0	0	+0.06	0	-.22	0
	30			1.1892	0	0		
45	22	0	0	0	+0.11	0	-.23	0
	30			1.1892	0	0		
46	22	232.1429	232.1429	0	+0.12	0	-.25	-58.036
	30			1.1892	0	0		
Σ			969.334			0		-236.468

x_{jo}, $(z_{n+i} - c_{n+i})$, da_{ij}/dt_h, and dc_j/dt_h for a -0.30 change in average daily gain where j is defined as before.

Substituting elements from table 4.30 into equation 3.51, the following economic value for average daily gain is found:

For a +0.30 change in average daily gain

$$3.51 \qquad E.V. = \left[\frac{1}{969.334}\right] [0 + 163.585] = \$.17$$

Substituting elements from table 4.31 into equation 3.51, the following economic value for average daily gain is found:

For a -0.30 change in average daily gain

$$3.51 \qquad E.V. = \left[\frac{1}{969.334}\right] [0 + (-236.468)] = \$-.24$$

5. Changes in the optimal basis

The derivation of the economic values of each of the three traits, backfat, feed efficiency, and average daily gain, using the revised computable form, have been done under the assumption that each change of each trait was small enough such that the optimal basis would not change. The question of whether or not the optimal basis does change, due to the changes of the traits, has not been discussed.

A procedure that may be used to find whether or not the optimal basis does change with a change in a trait is to solve a "new" linear program. The "new" linear program is actually identical to the initial linear program with the exception of altering relevant coefficients of the initial linear program. By adding the changes of the linear program coefficients that reflect a change in the h-th trait to respective coefficients of the initial linear program, the "new" linear program is formed.

So that it be known whether or not the revised computable form should have been used in deriving the economic values for each of the traits, 12 new linear programs -- four for each trait -- were developed and solved to see if the optimal mix of activities changed with changes in the traits. From earlier discussions on the computable form, it is known that if the optimal mix of activities changes with a change in the h-th trait, the revised computable form should not be used to derive the economic value of the h-th trait because the revised computable form will give an inaccurate economic value.

In only two of the 12 new linear programs were the optimal feasible bases different from the optimal feasible basis in table 4.11a. The two programs having different optimal feasible bases were for -.15 and -.30 changes in average daily gain. Activities A_{31} and A_{51} were not in the optimal basis in table 4.11a. They are in the optimal bases for the two programs reflecting reductions in average daily gain. Variables having values in these two solutions that differ from their values in table 4.11a are shown in table 4.32. An optimal basis that differs from the optimal basis of the initial linear program indicates that a different procedure from the revised computable form must be used to derive the economic value of the trait.

Because the optimal mix of activities changed with each of the negative changes in average daily gain, the economic values derived for average daily gain using the revised computable form will

Table 4.32. Portions of optimal mixes of real activities that changed due to changes in the traits: Model I[a]

Activity j	Initial optimal basis x_{jo}	-0.15 change in ADG x_{jo}	-0.30 change in ADG x_{jo}
A13	1.8158	1.8158	0.2968
A17	26.3158	26.3158	24.7968
A21	25.0	25.0	23.557
A25	197.50	197.50	186.10
A29	270.8333	71.002	151.1489
A31	0.0	184.46	110.478
A38	195.525	195.525	184.239
A49	589.875	154.642	329.202
A51	0.0	474.80	284.370
A58	387.14	187.14	364.794
A69	3.2895	3.2895	3.0996
A71	100.0	100.0	94.228
A76	92.6333	77.262	83.427

[a]The total optimal basis of Model I is shown in table 4.11a.

be inaccurate. Thus, a new procedure must be developed for these changes so as to derive the economic values.

Assume that the maximum value of the objective function of the initial linear program is written

4.8 $$Z_o = \sum_{i \epsilon B_o} c_i \, x_{io}$$

where $\sum_{i \epsilon B_o}$ denotes summation over all variables in the optimal basis of the optimal solution of the initial linear program

Assume that the maximum value of the objective function of the "new" linear program is written

4.9 $$Z' = \sum_{i \in B'} c_i x_{io}$$

where $\sum_{i \in B'}$ denotes summation over all variables in the optimal basis of the optimal solution of the "new" linear program

Now, the new equation that can be used to derive economic values of the h-th trait for changes that change the optimal mix of activities is written

4.10 $$E.V. = \left[\frac{2}{\sum_{j*} x_{j*} + \sum_{j*} x'_{j*}}\right] [Z_o - Z']$$

where E.V. is the economic value

$\sum_{j*} x_{j*}$ is the number of animals produced by the farm firm in the initial linear program and where j* identifies an activity that produces an animal that will have a unit improvement in the h-th trait

$\sum_{j*} x'_{j*}$ is the number of animals produced by the farm firm in the new linear program with the h-th trait improved and where j* identifies an activity that produces an animal that has a unit improvement in the h-th trait

(Note: In the case where the optimal basis does not change or $\Sigma x_{j*} = \Sigma x'_{j*}$, equation 4.10 can still be used. In such a case, equations 3.51 and 4.10 yield the same economic value.)

Using information from the optimal solutions of the "new" linear programs that include changes in average daily gain, table 4.33 can be constructed. Table 4.33 shows the value of the objective function for the initial and "new" linear program solutions and also the number of animals produced by the farm firm with the h-th trait improved.

Table 4.33. Elements in equation 4.10 needed to find the economic values of average daily gain for -0.15 and -0.30 changes in average daily gain due to the inability to find the economic values using the revised computable form

Initial program		"New" programs: -0.15 change in ADG		"New" programs: -0.30 change in ADG	
Z_o	$\sum_{j*} x_{j*}$	Z'	$\sum_{j*} x_{j*}'$	Z'	$\sum_{j*} x_{j*}'$
23,204.24	969.334	23,002.25	953.963	22,752.54	948.842

Substituting elements from table 4.33 into equation 4.10, the following economic values for average daily gain are found:

For a -0.15 change in average daily gain

$$4.10 \qquad \text{E.V.} = \left[\frac{2}{\sum_{j*} x_{j*} + \sum_{j*} x_{j*}'}\right] [Z' - Z_o]$$

$$= \left[\frac{2}{969.334} + 953.963\right] [23{,}002.25 - 23{,}204.24]$$

$$= [0.001039] [-201.99]$$

$$= \$-.21$$

This value is nearly two times larger than the \$-.11 value found when inappropriately using the revised computable form.

For a -0.30 change in average daily gain

$$4.10 \qquad E.V. = [\frac{2}{\sum_{j*} x_{j*} + \sum_{j*} x'_{j*}}] [Z' - Z_o]$$

$$= [\frac{2}{969.334} + 948.842] [22,752.54 - 23,204.24]$$

$$= [0.001042] [-451.70]$$

$$= \$-.47$$

This value is also nearly two times larger than the \$-.24 value found when inappropriately using the revised computable form. Note: Although the absolute economic values found by using equation 4.10 are larger than those found by equation 3.51, this is not always true as will be seen later.

D. Summary

The proposed method by which economic values of traits may be found was presented in this chapter. First, a hypothetical, but realistic, farm firm was developed. Using some of the information from the hypothetical farm firm, a linear program of the farm firm was developed by forming basic parameters of the linear program to reflect the farm firm. After solving for the optimal feasible solution of the linear program of the farm firm, the revised computable form was used to find the economic values of backfat, feed efficiency, and average daily gain, given the change in each respective trait.

Following the demonstration of deriving economic values of traits, using the revised computable form, it was demonstrated that the revised computable form does not determine the correct economic value of a trait given a change that is too large in the trait, due to the change in the optimal mix of activities of the optimal feasible solution. But, by obtaining optimal feasible solutions for new linear programs that reflect the respective changes in traits, economic values can be derived by finding the difference between the maximum value of the objective function of the new linear program and the maximum value of the objective function of the initial linear program and dividing that difference by the average number of animals with trait changes between the two linear programs.

The economic values derived by using the revised computable form for backfat, feed efficiency, and +0.15 and +0.30 changes in average daily gain, and the procedure of solving a new optimal feasible solution for -0.15 and -0.30 changes in average daily gain are presented in table 4.34.

Table 4.34. Economic values for backfat, feed efficiency, and average daily gain: Model I

Trait	Change of the trait			
	$+1\sigma$[a]	-1σ	$+2\sigma$	-2σ
Backfat	$ -.95[b]	$ .96[b]	$-1.90[b]	$1.91[b]
Feed efficiency	-1.44[b]	1.44[b]	-2.88[b]	2.88[b]
Average daily gain	.09[c]	-.21[d]	.17[c]	-.47[d]

[a]The symbol σ represents standard deviation.

[b]Value was derived using equation 3.51(a).

[c]Value was derived using equation 3.51.

[d]Value was derived using equation 4.10.

One additional comment can be made in summarizing this chapter. Since the only way to determine if the optimal mix of activities does change (upon changing linear program coefficients so as to reflect a change in a trait) is by solving for an optimal feasible solution to a new linear program, it may be advantageous to exclude the use of the revised computable form from the derivation process. Yet, if one is sure the change in the trait is small enough so as not to change the optimal mix of activities, as was the case with illustrations of backfat, feed efficiency, and +0.15 and +0.30 changes in average daily gain, the revised computable form is an excellent tool to use in deriving economic values of traits.

V. EFFECTS OF VARYING CONDITIONS ON ECONOMIC VALUES

"... Economic values of traits may vary with the particular locality or nature of the enterprise ..." [Hazel, 15, p. 487].

Geographic locations cause great variations in farm firm enterprises due to differences in climatic conditions, management practices, etc., that may be typical for a certain area. Also, with the numerous levels of technology available in livestock production, no two livestock enterprises are exactly alike. It therefore stands to reason that economic values of traits vary with locality and nature of the enterprise.

Examining the quotation from Hazel [15] in a little more detail, it can be seen that the quotation has been discussed, in part, earlier. The particular locality of the enterprise was mentioned in section IV. A.2.b. Related environmental conditions such as climatic conditions, management, and geographic conditions, were assumed to be typical for a Midwest swine farm in developing Model I. The locality must be indicated through an assumption so as to specify the particular locality to which the derived economic value is applicable.

The nature of the enterprise was described in sections IV.A.1 and IV.A.2.a. The nature of the enterprise was partially described by the general description of the swine farm (section IV.A.1). Looking at the number of farrowings per year, whether or not feeder pigs are purchased, how the gilts and sows are supplied for farrowings, etc., indicates the nature of the enterprise. The technology of the farm enterprise also partially describes the nature of the enterprise (section IV.A.1.a). By analyzing the methods and facilities used in production of output of the enterprise of the farm firm, the nature of the enterprise is viewed. It can also be seen, then, that the nature of the enterprise must be indicated so as to specify the nature of the enterprise to which the derived economic value is applicable.

So as to demonstrate that the quotation from Hazel [15] is true, new economic values will be derived in the following sections. By altering the RHS values of Model I, to obtain "revised Model I," the fact that economic values of traits may vary with the "particular locality" of the enterprise will be demonstrated. By developing a new linear program of a different swine enterprise (or swine farm), Model II, the fact that economic values of traits may vary with the "nature of the enterprise" will be demonstrated.

A. Changing RHS Values of Model I

Certain localities may consider a working day to be different than in other localities, simply because of the number of daylight hours. Other localities may differ in total labor hours available because of an unwillingness to spend more than a certain number of hours working on a certain farm firm enterprise. As a result, different localities may be reflected by assuming a different number of hours available for labor.

Assume that the management is unwilling to spend as many hours working with the swine farm during the cropping months as listed in table 4.2a. As a result, the RHS values for available labor of

Model I are changed to new values and a new locality is considered for the swine farm.

1. Different RHS values

The new RHS values for available labor in the new locality are shown in table 5.1. All other RHS values of the linear program of the two swine farms are the same. Table 5.1 also shows the RHS values for available labor of the Midwest swine farm.

Table 5.1. A comparison of RHS values for available labor of the Midwest swine farm and the swine farm of a different locality

Row number	Month	Available hours of the Midwest swine farm	Available hours of the swine farm of a different locality
1	November 1972	160	140
2	December 1972	196	196
3	January 1973	216	216
4	February 1973	192	192
5	March 1973	198	198
6	April 1973	160	140
7	May 1973	160	140
8	June 1973	160	140
9	July 1973	216	216
10	August 1973	208	208
11	September 1973	168	168
12	October 1973	160	140
13	November 1973	160	140
14	December 1973	196	196
15	January 1974	216	216
16	February 1974	192	192
17	March 1974	198	198
18	April 1974	160	140
19	May 1974	160	140
20	June 1974	160	140
21	July 1974	216	216
22	August 1974	208	208

2. The optimal solution

Since the two linear programs of the two swine farms are identical with the exception of the labor availability during the cropping months, the optimal feasible solution of the linear program of the revised Model I is easily found by using the process discussed in section III.D.2 after changing the relevant RHS values. The optimal feasible solution of the revised Model I is shown in table 5.2a, table 5.2b, and table 5.2c.

3. Sensitivity analysis

Once the optimal feasible solution of the linear program of the revised Model I is found, new economic values for the traits are also ready to be found. In sections IV.C.1.b, IV.C.2.b, and IV.C.3.b, changes of linear program coefficients that reflect changes in backfat, feed efficiency, and average daily gain, respectively, were found. These same changes of the linear program coefficients are used with information from the optimal feasible solution of the revised Model I to derive the new economic values. This can be done since the two linear programs are identical except for relevant changes in certain RHS values.

a. Backfat Changes in linear program coefficients that reflect the changes in backfat were shown in tables 4.16 and 4.28. Given table 5.3, where the changes in the linear program coefficients are given with relevant information from the optimal feasible solution of the linear program of the revised Model I, using equation 3.51(a), new economic values are found.

Table 5.2a. Optimal mix of real activities and their shadow prices: revised Model I

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Purchase gilts to farrow in May	A01	--	-38.3300
Purchase gilts to farrow in August	A02	--	-61.6900
Purchase gilts to farrow in November	A03	--	-129.9927
Purchase gilts to farrow in February	A04	--	-135.4000
Purchase boar to service females	A05	2 boars	--
Prepare purchased gilts for breeding and farrowing in May	A06	--	--
Prepare purchased gilts for breeding and farrowing in August	A07	--	--
Prepare purchased gilts for breeding and farrowing in November	A08	--	--
Prepare purchased gilts for breeding and farrowing in February	A09	--	--
Feed boars	A10	2 boars	--
Raise gilts to farrow in May	A11	26.3158 gilts	--
Raise gilts to farrow in August	A12	26.3158 gilts	--
Raise gilts to farrow in November	A13	--	-46.2327
Raise gilts to farrow in February	A14	1.8158 gilts	--
Prepare breeding herd for breeding and farrowing in May	A15	26.3158 gilts	--

Table 5.2a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Prepare breeding herd for breeding and farrowing in August	A16	26.3158 gilts	--
Prepare breeding herd for breeding and farrowing in November	A17	24.5000 sows	--
Prepare breeding herd for breeding and farrowing in February	A18	26.3158 sows	--
Farrowing in May	A19	25.0000 gilts	--
Farrowing in August	A20	25.0000 gilts	--
Farrowing in November	A21	23.2750 sows	--
Farrowing in February	A22	25.0000 sows	--
Feed weaned May pigs to 40 pounds	A23	180.0000 pigs	--
Feed weaned August pigs to 40 pounds	A24	180.0000 pigs	--
Feed weaned November pigs to 40 pounds	A25	183.8725 pigs	--
Feed weaned February pigs to 40 pounds	A26	197.5000 pigs	--
Feed 40 pound pigs farrowed in May to 180 pounds	A27	--	-2.7441
Feed 40 pound pigs farrowed in May to 200 pounds	A28	--	-2.0100
Feed 40 pound pigs farrowed in May to 220 pounds	A29	270.8333 hogs	--
Feed 40 pound pigs farrowed in May to 240 pounds	A30	--	-4.1376
Feed 40 pound pigs farrowed in May to 260 pounds	A31	--	--
Feed 40 pound pigs farrowed in August to 180 pounds	A32	--	-10.6828

Table 5.2a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Feed 40 pound pigs farrowed in August to 200 pounds	A33	--	-6.5125
Feed 40 pound pigs farrowed in August to 220 pounds	A34	--	-4.61858
Feed 40 pound pigs farrowed in August to 240 pounds	A35	--	-2.4355
Feed 40 pound pigs farrowed in August to 260 pounds	A36	234.2912 hogs	--
Feed 40 pound pigs farrowed in November to 180 pounds	A37	--	-.5560
Feed 40 pound pigs farrowed in November to 200 pounds	A38	182.0338 hogs	--
Feed 40 pound pigs farrowed in November to 220 pounds	A39	--	-1.2946
Feed 40 pound pigs farrowed in November to 240 pounds	A40	--	-4.1038
Feed 40 pound pigs farrowed in November to 260 pounds	A41	--	-8.1654
Feed 40 pound pigs farrowed in February to 180 pounds	A42	--	-7.7143
Feed 40 pound pigs farrowed in February to 200 pounds	A43	--	-7.6924
Feed 40 pound pigs farrowed in February to 220 pounds	A44	--	-7.8657
Feed 40 pound pigs farrowed in February to 240 pounds	A45	--	-6.3934
Feed 40 pound pigs farrowed in February to 260 pounds	A46	232.1429 hogs	--
Market May farrowed 180 pound market hogs	A47	--	--
Market May farrowed 200 pound market hogs	A48	--	--
Market May farrowed 220 pound market hogs	A49	589.8750 cwt.	--

Table 5.2a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_j	Income penalty (z_j-c_j)
Market May farrowed 240 pound market hogs	A50	--	--
Market May farrowed 260 pound market hogs	A51	--	-1.1099
Market August farrowed 180 pound market hogs	A52	--	--
Market August farrowed 200 pound market hogs	A53	--	--
Market August farrowed 220 pound market hogs	A54	--	--
Market August farrowed 240 pound market hogs	A55	--	--
Market August farrowed 260 pound market hogs	A56	603.0655 cwt.	--
Market November farrowed 180 pound market hogs	A57	--	--
Market November farrowed 200 pound market hogs	A58	360.4269 cwt.	--
Market November farrowed 220 pound market hogs	A59	--	--
Market November farrowed 240 pound market hogs	A60	--	--
Market November farrowed 260 pound market hogs	A61	--	--
Market February farrowed 180 pound market hogs	A62	--	--
Market February farrowed 200 pound market hogs	A63	--	--
Market February farrowed 220 pound market hogs	A64	--	--
Market February farrowed 240 pound market hogs	A65	--	--
Market February farrowed 260 pound market hogs	A66	597.5357 cwt.	--

Table 5.2a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty $(z_j - c_j)$
Market non-conceived gilts in January	A67	3.2895 cwt.	--
Market non-conceived gilts in April	A68	3.2895 cwt.	--
Market non-conceived gilts in July	A69	3.0625 cwt.	--
Market non-conceived gilts in October	A70	3.2895 cwt.	--
Market sows after November farrowing	A71	93.1000 cwt.	--
Market sows after February farrowing	A72	100.0000 cwt.	--
Market boars in November 1973	A73	8.0000 cwt.	--
Market gilts culled after first farrowing (May)	A74	0.8250 cwt.	--
Market gilts culled after first farrowing (August)	A75	0.8250 cwt.	--
Purchase 40 pound feeder pigs in June	A76	92.6333 pigs	--
Purchase 40 pound feeder pigs in September	A77	56.0912 pigs	--
Purchase 40 pound feeder pigs in December	A78	--	-6.1392
Purchase 40 pound feeder pigs in March	A79	36.6179 pigs	--

Table 5.2b. Income over variable costs, Z_o: revised Model I

	Amount
Income	$22,405.39

Substituting elements from table 5.3 into equation 3.51(a), the following new economic values for backfat are found:

For a +0.15 change in backfat

3.51(a) $$E.V. = [\frac{1}{910.107}] [-866.107] = \$-.95$$

For a -0.15 change in backfat

3.51(a) $$E.V. = [\frac{1}{910.107}] [872.215] = \$.96$$

For a +0.30 change in backfat

3.51(a) $$E.V. = [\frac{1}{910.107}] [-1728.741] = \$-1.90$$

For a -0.30 change in backfat

3.51(a) $$E.V. = [\frac{1}{910.107}] [1734.848] = \$1.91$$

b. Feed efficiency Changes in linear program coefficients that reflect the changes in feed efficiency were shown in tables 4.20 and 4.29. Given table 5.4, where the changes in the linear program coefficients are given with relevant information from the optimal feasible solution of the revised Model I, using equation 3.51(a), new economic values are found.

Table 5.2c. Fixed input use and each fixed input's shadow price: revised Model I

Fixed input	Row name	Row (constraint) number i	Amount available a_{io}	Amount used ($a_{io}-x_{n+i}$)	Marginal value product ($z_{n+i}-c_{n+i}$)
November 1972 labor	M01	1	140	12.282	--
December 1972 labor	M02	2	196	11.819	--
January 1973 labor	M03	3	216	20.404	--
February 1973 labor	M04	4	192	27.069	--
March 1973 labor	M05	5	198	24.806	--
April 1973 labor	M06	6	140	69.904	--
May 1973 labor	M07	7	140	91.740	--
June 1973 labor	M08	8	140	96.071	--
July 1973 labor	M09	9	216	122.447	--
August 1973 labor	M10	10	208	129.402	--
September 1973 labor	M11	11	168	128.800	--
October 1973 labor	M12	12	140	140.000	83.394
November 1973 labor	M13	13	140	122.573	--
December 1973 labor	M14	14	196	117.631	--
January 1974 labor	M15	15	216	119.550	--
February 1974 labor	M16	16	192	122.902	--
March 1974 labor	M17	17	198	101.117	--
April 1974 labor	M18	18	140	59.116	--
May 1974 labor	M19	19	140	32.500	--
June 1974 labor	M20	20	140	32.500	--
July 1974 labor	M21	21	216	32.500	--
August 1974 labor	M22	22	208	18.249	--
May 1973 farrowing capacity	F01	23	25	25.000	38.121
August 1973 farrowing capacity	F02	24	25	25.000	97.930
November 1973 farrowing capacity	F03	25	25	23.275	--
February 1974 farrowing capacity	F04	26	25	25.000	57.366
Building #1 finishing capacity	R01	27	3250	3250.000	0.797
Building #2 finishing capacity	R02	28	3250	2811.494	--
Building #1 finishing capacity	R03	29	3250	2093.388	--
Building #2 finishing capacity	R04	30	3250	3250.000	1.189
Boar equality	R25	35	2	2.000	-280.557

Table 5.3. Elements in equation 3.51(a) needed to find the economic values of backfat for +0.15, -0.15, +0.30, and -0.30 changes in backfat: revised Model I

			+0.15 change in backfat		-0.15 change in backfat	
j	x_{jo}	x_j	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
47	0	0	-.41	0	+.43	0
48	0	0	-.42	0	+.42	0
49	589.875	268.125	-.43	-253.646	+.42	247.748
50	0	0	-.42	0	+.41	0
51	0	0	-.42	0	+.42	0
52	0	0	-.40	0	+.42	0
53	0	0	-.40	0	+.41	0
54	0	0	-.41	0	+.41	0
55	0	0	-.41	0	+.40	0
56	603.0655	231.948	-.40	-241.226	+.41	247.257
57	0	0	-.39	0	+.41	0
58	360.4269	180.213	-.40	-144.171	+.40	144.171
59	0	0	-.40	0	+.39	0
60	0	0	-.39	0	+.40	0
61	0	0	-.40	0	+.40	0
62	0	0	-.37	0	+.39	0
63	0	0	-.39	0	+.38	0
64	0	0	-.38	0	+.39	0
65	0	0	-.38	0	+.38	0
66	597.5357	229.821	-.38	-227.064	+.39	233.039
Σ		910.107		-866.107		872.215

Table 5.3. Continued

j	+0.30 change in backfat		-0.30 change in backfat	
	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
47	-.83	0	+.85	0
48	-.84	0	+.84	0
49	-.85	-501.394	+.84	495.495
50	-.84	0	+.84	0
51	-.84	0	+.84	0
52	-.81	0	+.82	0
53	-.81	0	+.82	0
54	-.82	0	+.81	0
55	-.81	0	+.81	0
56	-.81	-488.483	+.82	494.514
57	-.78	0	+.79	0
58	-.79	-284.737	+.79	284.737
59	-.79	0	+.79	0
60	-.78	0	+.80	0
61	-.79	0	+.79	0
62	-.76	0	+.77	0
63	-.77	0	+.76	0
64	-.77	0	+.77	0
65	-.76	0	+.77	0
66	-.76	-454.127	+.77	460.102
Σ		-1728.741		1734.848

Table 5.4. Elements in equation 3.51(a) needed to find the economic values of feed efficiency for +0.15, -0.15, +0.30, and -0.30 changes in feed efficiency: revised Model I

			+0.15 change in feed efficiency		-0.15 change in feed efficiency	
j	x_{jo}	x_j	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
27	0	0	$-1.08	0	$+1.08	0
28	0	0	-1.22	0	+1.24	0
29	270.8333	270.8333	-1.37	-371.042	+1.38	373.750
30	0	0	-1.53	0	+1.53	0
31	0	0	-1.68	0	+1.68	0
32	0	0	-1.05	0	+1.05	0
33	0	0	-1.19	0	+1.19	0
34	0	0	-1.34	0	+1.34	0
35	0	0	-1.48	0	+1.49	0
36	234.2911	234.2911	-1.63	-381.894	+1.63	381.894
37	0	0	-1.02	0	+1.02	0
38	182.0338	182.0338	-1.16	-211.159	+1.16	211.159
39	0	0	-1.30	0	+1.30	0
40	0	0	-1.44	0	+1.44	0
41	0	0	-1.58	0	+1.58	0
42	0	0	-.99	0	+.99	0
43	0	0	-1.12	0	+1.12	0
44	0	0	-1.26	0	+1.26	0
45	0	0	-1.40	0	+1.40	0
46	232.1429	232.1429	-1.53	-355.179	+1.54	357.500
Σ		919.301		-1319.274		1324.303

Table 5.4. Continued

j	+0.30 change in feed efficiency		-0.30 change in feed efficiency	
	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
27	$-2.17	0	$+2.16	0
28	-2.46	0	+2.47	0
29	-2.75	-744.792	+2.76	747.500
30	-3.06	0	+3.06	0
31	-3.36	0	+3.36	0
32	-2.09	0	+2.09	0
33	-2.38	0	+2.39	0
34	-2.67	0	+2.68	0
35	-2.97	0	+2.97	0
36	-3.26	-763.789	+3.26	763.789
37	-2.03	0	+2.03	0
38	-2.31	-420.498	+2.31	420.498
39	-2.60	0	+2.60	0
40	-2.88	0	+2.88	0
41	-3.16	0	+3.15	0
42	-1.97	0	+1.98	0
43	-2.24	0	+2.25	0
44	-2.52	0	+2.52	0
45	-2.79	0	+2.80	0
46	-3.06	-710.357	+3.07	712.679
Σ		-2639.436		2644.466

Substituting elements from table 5.4 into equation 3.51(a), the following new economic values for feed efficiency are found:

For a +0.15 change in feed efficiency

$$3.51(a) \qquad E.V. = [\frac{1}{919.301}] [-1319.274] = \$-1.44$$

For a -0.15 change in feed efficiency

$$3.51(a) \qquad E.V. = [\frac{1}{919.301}] [1324.303] = \$1.44$$

For a +0.30 change in feed efficiency

$$3.51(a) \qquad E.V. = [\frac{1}{919.301}] [-2639.436] = \$-2.87$$

For a -0.30 change in feed efficiency

$$3.51(a) \qquad E.V. = [\frac{1}{919.301}] [2644.466] = \$2.88$$

c. Average daily gain Changes in linear program coefficients that reflect the changes in average daily gain were shown in tables 4.26, 4.27, 4.30, and 4.31. Each of these respective changes shown in the tables cause changes in the optimal mix of activities of the optimal feasible solution of revised Model I. As a result, it is necessary to use equation 4.10 to derive the new economic values.

Using information from the optimal solutions of the linear programs reflecting +0.15, -0.15, +0.30, and -0.30 changes in average daily gain, a table of needed information for equation 4.10 can be constructed. Table 5.5 shows the value of the objective function for the initial

Table 5.5. Elements in equation 4.10 needed to find the economic values of average daily gain for +0.15, -0.15, +0.30, and -0.30 changes in average daily gain: revised Model I

Initial program		"New" program: +0.15 change in ADG		-0.15 change in ADG		+0.30 change in ADG		-0.30 change in ADG	
Z_o	$\sum_{j*} x_{j*}$	Z'	$\sum_{j*} x'_{j*}$	Z'	$\sum_{j*} x'_{j*}$	Z'	$\sum_{j*} x'_{j*}$	Z'	$\sum_{j*} x'_{j*}$
22,405.39	919.301	23,294.19	969.334	21,493.85	874.798	23,367.83	969.334	21,179.05	871.815

and "new" linear program solutions and also the number of animals produced by the farm firm with the h-th trait improved.

Substituting elements from table 5.5 into equation 4.10, the following new economic values for average daily gain are found:

For a +0.15 change in average daily gain

$$4.10 \qquad E.V. = \left[\frac{2}{919.301 + 969.334}\right] [23{,}294.19 - 22{,}405.39]$$

$$= \$.94$$

Inappropriately using equation 3.51 to derive the economic value of average daily gain for a +0.15 change in average daily gain would have given an economic value of $1.08. This is a case where the absolute economic value found by using equation 4.10 is not larger than the one found by using equation 3.51 when the basis changed.

For a -0.15 change in average daily gain

$$4.10 \qquad E.V. = \left[\frac{2}{919.301 + 874.798}\right] [21{,}493.85 - 22{,}405.39]$$

$$= \$-1.02$$

Inappropriately using equation 3.51 to derive the economic value of average daily gain for a -0.15 change in average daily gain would have given an economic value of $-1.58. This is another case where the absolute economic value found by using equation 4.10 is not larger than the absolute economic value found by using equation 3.51 when the basis changed. There are other examples also in the remaining thesis, but they will not be pointed out.

For a +0.30 change in average daily gain

$$4.10 \qquad E.V. = \left[\frac{2}{919.301 + 969.334}\right] [23,367.83 - 22,405.39]$$

$$= \$1.02$$

Inappropriately using equation 3.51 to derive the economic value of average daily gain for a +0.30 change in average daily gain would have given an economic value of $1.89.

For a -0.30 change in average daily gain

$$4.10 \qquad E.V. = \left[\frac{2}{919.301 + 871.815}\right] [21,179.05 - 22,405.39]$$

$$= \$-1.37$$

Inappropriately using equation 3.51 to derive the economic value of average daily gain for a -0.30 change in average daily gain would have given an economic value of $-1.72.

4. Comparison of economic values

Briefly comparing the economic values derived from Model I and revised Model I, it can be seen that the economic values differ significantly for average daily gain. Looking at table 5.6, revised Model I has significantly larger absolute economic values for average daily gain than Model I. It also can be seen that the economic values for backfat and feed efficiency are relatively the same for both swine farms.

It is important to remember, though, this is a demonstration of how the economic values may vary because of a particular locality.

Table 5.6. A comparison of economic values of backfat, feed efficiency, and average daily gain for Model I and revised Model I

Model farm	Change	Trait		
		Backfat	Feed efficiency	Average daily gain
Model I	+1σ	$ -.95	$-1.44	$ +.09
	-1σ	+.96	+1.44	-.21
	+2σ	-1.90	-2.88	+.17
	-2σ	+1.91	+2.88	-.47
Revised Model I	+1σ	-.95	-1.44	+.94
	-1σ	+.96	+1.44	-1.02
	+2σ	-1.90	-2.87	+1.02
	-2σ	+1.91	+2.88	-1.37

It can therefore only be concluded that economic values may vary because of a particular locality.

B. A New Linear Program - Model II

As was indicated earlier, by developing a new linear program of a different swine enterprise (or swine farm), Model II, the fact that economic values of traits may "vary with the nature of the enterprise" can be demonstrated. The number of farrowings per year, whether or not feeder pigs are purchased, whether gilts are purchased or raised, the methods and facilities used in production, etc., are all characteristics describing the "nature of the enterprise."

Sections IV.A.1 and IV.A.1.a described the "nature of the swine enterprise" for Model I. By altering the assumptions and descriptions in these sections, Model II can easily be developed.

1. Model II

a. General description of the swine farm The swine farm that is developed in this section is flexible. The swine farm has two farrowing activities and two feeder pig buying activities. Farrowing times are in April and October. Feeder pigs are purchased in May and November.

As in Model I, the females that farrow may come from various sources. The swine farm has the option of purchasing new gilts or raising gilts for each farrowing. The gilts that farrow in April will be allowed to farrow again in October, though, provided they are not culled. Gilts that do not conceive or are culled prior to the second farrowing are marketed and may be replaced by newly purchased or raised gilts. Females that farrow in October are marketed following the weaning of their pigs. Gilts that do not conceive for farrowing in April are also marketed.

One boar will be purchased in October to breed the gilts and sows that farrow in April and October. In the following October, the boar is marketed, having served his purpose.

As in Model I, the swine farm in Model II will feed purchased and/or farrowed pigs to weights of 180, 200, 220, 240, or 260 pounds. Since there are only two farrowings and two possible times to purchase feeder pigs, and also five possible market weights, there will be only 10 possible times to market finished hogs.

Other activities of the swine farm are included in the swine farm of Model II as in Model I. These activities, as before, are partially dependent upon the basic assumptions of the swine farm.

The assumptions are looked at in closer detail in the following section V.B.2.

b. Assumptions Since the differences between the swine farms of Model I and Model II are due to the "nature of the enterprises," certain basic assumptions between the two linear programs remain the same. Other assumptions will change due to the nature of each enterprise. The following assumptions indicate the differences that lie between the swine farms of Model I and Model II.

(1) Technology (Assumption 1) The swine farm uses a pasture farrowing system. Portable "A" frame houses are used as housing for the sows and litters. Each "A" frame house is assumed to house one sow and her litter. Self-feeders are used to feed the sows with the sows having access to the self-feeders for limited periods of time each day. Water is assumed to be piped to the pasture.

The swine farm also has a partial confinement growing-finishing unit available which is identical to those of the swine farm in Model I.

(2) Environmental conditions (Assumption 2) The environmental conditions of which the swine farm of Model II is subject to are identical to those of which the swine farm of Model I were subject.

(3) Period length (Assumption 3) The length of time that was assumed in developing the linear program of the swine farm of Model II was a 19 month period, beginning October 1, 1972, and ending April 31, 1974. The 19 month period represents the time period in which sequential activities associated with a swine farm (i.e.,

purchasing gilts that farrow through marketing slaughter hogs) farrowing two litters could occur.

(4) Discounting to present value (Assumption 4) The opportunity cost or discount rate used in the discounting procedure is assumed to be 12 percent per annum or 1 percent per month for the linear program of Model II as in Model I. The 12 percent rate of discount is assumed to be the average rate of return on essentially riskless investments covering any rate of pure time preference and rate of inflation during the 19 month period.

As was done in the discounting procedure in developing the linear program of Model I, the net returns of the activities in the linear program of Model II are discounted to present value as of November 1, 1972. Since the net returns of activities in both linear programs are discounted to present value as of November 1, 1972, the economic values derived from both models can be compared, irregardless of the periods not being exactly the same.

(5) Current stage of genetic progress (Assumption 5) The current stage of genetic progress of which the swine farm of Model II is assumed to have is identical to that of which the swine farm of Model I was assumed to have. These were shown in table 4.1.

(6) Fixed inputs available (Assumption 6) The availability of fixed inputs for the swine farm of Model II is identical to that of the swine farm of Model I with a few changes. The labor availability is the same except that there is labor available in October 1972, which amounts to 160 hours, and labor in May, June, July, and August of 1974 is not needed for swine. The farrowing capacities

are the same except they are needed in April and October and not in May, August, November, and February, and also, the farrowing capacities require pasture, not a central farrowing house. Only one partial confinement growing-finishing house is needed, not two, but the available area per house is the same. Finally, only one boar is required to be purchased, not two as in the swine farm of Model I.

(7) Rations (Assumption 7) The rations fed by the swine farm of Model II are identical to those which are fed by the swine enterprise of Model I. The rations were shown in tables 4.3a through 4.3d.

(8) Prices (Assumption 8) The prices assumed in developing the linear program of the swine farm of Model II are very similar to those prices shown in tables 4.5a and 4.5b. The differences in the assumed prices occur because of differences in purchasing and marketing times. The assumed prices for Model II are shown in tables 5.7a and 5.7b.

c. Formation of the linear program coefficients The formation of the linear program coefficients was handled in the same manner as was described in section IV.A.3. Many of the linear program coefficients of the two linear programs were the same. Some of them, though, were different. Those coefficients that were different were different because of altering some of the assumptions made so as to develop the linear programs. Altering certain assumptions caused the nature of the enterprises to vary.

d. Specific description of the linear program of the swine farm The linear program of Model II is smaller (i.e., fewer rows

Table 5.7a. Price assumptions for variable inputs: Model II

Input	Price	Input	Price
Corn	$ 2.20/bu.	Boar	$275
Soybean oilmeal	.12/lb.	Group #1 feeder pigs	28.56/head
Dicalcium phosphate	.10/lb.	Group #2 feeder pigs	31.66/head
Limestone	.02/lb.	Transportation:	
Salt	.025/lb.	Purchased gilts	5/head
Trace mineral premix	.10/lb.	Purchased boar	5/head
Vitamin premix	.60/lb.	Purchased feeder pigs	1/head
Dried whey	.09/lb.	Market hogs	2/cwt.
Tylosin	.12/gm	Non-breeder gilts	2/cwt.
ASP-250	.033/gm	Culled gilts	2/cwt.
Furazolidone	.06/gm	Market sows	2/cwt.
Group #1 purchased gilts	98.75	Market boar	2/cwt.
Group #2 purchased gilts	125.75		
Group #1 raised gilts	49.42		
Group #2 raised gilts	49.50		

Table 5.7b. Price assumptions for farm firm output: Model II

Output	Price	Output	Price
180 pound April hogs	$46.40/cwt.	Non-conceived Group #1 gilts	$27.15/cwt.
200 pound April hogs	46.88/cwt.	Non-conceived Group #2 gilts	41.02/cwt.
220 pound April hogs	44.50/cwt.	Culled gilts	33.15/cwt.
240 pound April hogs	42.67/cwt.	Market sows	39.22/cwt.
260 pound April hogs	43.31/cwt.	Market boar	32.00/cwt.
180 pound October hogs	38.98/cwt.		
200 pound October hogs	37.57/cwt.		
220 pound October hogs	34.82/cwt.		
240 pound October hogs	33.32/cwt.		
260 pound October hogs	32.36/cwt.		

and fewer columns) than the linear program of Model I. This is due to the fact that the swine farm of Model I has four farrowings during the time period whereas the swine farm of Model II has only two farrowings. Because the linear program of Model II is smaller than the linear program of Model I, the linear program of Model II can be shown in tableau form. The tableau is shown in appendix A, figure A.1.

The linear program tableau shown in appendix A, figure A.1, is interpreted in the same manner as the linear program shown in figures 4.2 and 4.3. The C-row indicates the c_j or net return coefficients of each activity. The RHS column indicates the a_{io} values or levels of fixed inputs. Finally, the coefficients within the C-row and RHS column borders are the a_{ij} coefficients or the input-output coefficients.

2. The optimal solution

As with the linear programs described earlier, the linear program of the pasture farrowing swine farm is found by using the process described in section III.D.2. The optimal feasible solution of the pasture farrowing swine farm linear program is found in tables 5.8a, 5.8b, and 5.8c.

3. Sensitivity analysis

As before, once the optimal feasible solution of the linear program is found, economic values for the traits are ready to be found. The economic values are found, though, only after finding the changes in the linear program coefficients that reflect the change in the h-th trait. The procedure to follow in finding changes in the linear program coefficients that reflect the change in the backfat, feed efficiency, and average daily gain traits was demonstrated in sections IV.C.1, IV.C.2, and IV.C.3, respectively.

Table 5.8a. Optimal mix of real activities and their shadow prices: Model II

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Purchase gilts to farrow in April	A01	--	-37.45
Purchase gilts to farrow in October	A02	--	-63.11
Purchase boar to service females	A03	1.0 boar	--
Prepare purchased gilts for breeding and farrowing in April	A04	--	--
Prepare purchased gilts for breeding and farrowing in Oct.	A05	--	--
Feed boar	A06	1.0 boar	--
Raise gilts to farrow in April	A07	26.3158 gilts	--
Raise gilts to farrow in Oct.	A08	1.8158 gilts	--
Prepare breeding herd for breeding and farrowing in April	A09	26.3158 gilts	--
Prepare breeding herd for breeding and farrowing in Oct.	A10	26.3158 sows	--
Farrowing in April	A11	25.0000 gilts	--
Farrowing in October	A12	25.0000 sows	--
Feed weaned April pigs to 40 pounds	A13	180.0000 pigs	--
Feed weaned October pigs to 40 pounds	A14	197.5000 pigs	--
Feed 40 pound pigs farrowed in April to 180 pounds	A15	--	-4.7409
Feed 40 pound pigs farrowed in April to 200 pounds	A16	270.8333 hogs	--
Feed 40 pound pigs farrowed in April to 220 pounds	A17	--	-0.8559
Feed 40 pound pigs farrowed in April to 240 pounds	A18	--	-3.3559
Feed 40 pound pigs farrowed in April to 260 pounds	A19	--	--
Feed 40 pound pigs farrowed in October to 180 pounds	A20	325.000 hogs	--
Feed 40 pound pigs farrowed in October to 200 pounds	A21	--	-0.1543
Feed 40 pound pigs farrowed in October to 220 pounds	A22	--	-3.3029
Feed 40 pound pigs farrowed in October to 240 pounds	A23	--	-5.6376

Table 5.8a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Feed 40 pound pigs farrowed in October to 260 pounds	A24	--	-6.8231
Market April farrowed 180 pound market hogs	A25	--	--
Market April farrowed 200 pound market hogs	A26	536.2500 cwt.	--
Market April farrowed 220 pound market hogs	A27	--	--
Market April farrowed 240 pound market hogs	A28	--	--
Market April farrowed 260 pound market hogs	A29	--	-0.0201
Market October farrowed 180 pound market hogs	A30	579.1500 cwt.	--
Market October farrowed 200 pound market hogs	A31	--	--
Market October farrowed 220 pound market hogs	A32	--	--
Market October farrowed 240 pound market hogs	A33	--	--
Market October farrowed 260 pound market hogs	A34	--	--
Market non-conceived gilts in December	A35	3.2895 cwt.	--
Market non-conceived gilts in June	A36	3.2895 cwt.	--
Market gilts culled after first farrowing (April)	A37	0.8250 cwt.	--
Market sows after November farrowing	A38	100.0000 cwt.	--
Market boar in October 1973	A39	4.0000 cwt.	--
Purchase 40 pound feeder pigs in May	A40	92.6333 pigs	--
Purchase 40 pound feeder pigs in November	A41	129.4750 pigs	--

Table 5.8b. Income over variable costs, Z_o: Model II

	Amount
Income	\$12,242.20

Table 5.8c. Fixed input use and each fixed input's shadow price: Model II

Fixed input	Row name	Row (constraint) number i	Amount available a_{io}	Amount used (a_{io} - x_{n+i})	Marginal value product (z_{n+i} - c_{n+i})
October 1972 labor	M01	1	160	12.062	--
November 1972 labor	M02	2	160	11.699	--
December 1972 labor	M03	3	196	37.534	--
January 1973 labor	M04	4	216	28.620	--
February 1973 labor	M05	5	192	24.870	--
March 1973 labor	M06	6	198	93.534	--
April 1973 labor	M07	7	160	142.151	--
May 1973 labor	M08	8	160	83.732	--
June 1973 labor	M09	9	160	91.037	--
July 1973 labor	M10	10	216	66.037	--
August 1973 labor	M11	11	208	65.495	--
September 1973 labor	M12	12	168	129.266	--
October 1973 labor	M13	13	160	144.370	--
November 1973 labor	M14	14	160	87.048	--
December 1973 labor	M15	15	196	65.000	--
January 1974 labor	M16	16	216	45.500	--
February 1974 labor	M17	17	192	48.750	--
March 1974 labor	M18	18	198	28.958	--
April 1974 labor	M19	19	160	25	--
April 1973 farrowing capacity	F01	20	25	25	76.915
October 1973 farrowing capacity	F02	21	25	25	182.498
Finishing capacity	R01	22	3250	3250	1.736
Finishing capacity	R02	23	3250	3250	.116
Boar equality	R19	26	1	1	-259.53

Economic values will be found in the following sections using the revised computable form (equation 3.51) and in certain cases, equation 4.10. The derivation of the changes of the linear program coefficients will not be presented since it is the same procedure as used in sections IV.C.1, IV.C.2, and IV.C.3. The changes of the linear program coefficients that reflect the change in the h-th trait will not be presented either, but are shown in appendix B.

a. Backfat Changes in linear program coefficients that reflect +0.15, -0.15, +0.30, and -0.30 changes in backfat are shown in appendix B, table B.1. The -0.15, +0.30, and -0.30 changes in backfat cause changes in the optimal mix of activities of the optimal feasible solution of each linear program reflecting these respective changes. The +0.15 change in backfat does not cause a change in the optimal mix of activities of the optimal feasible solution of the linear program reflecting the change. As a result, equation 3.51(a) is used to derive the new economic value of backfat with a +0.15 change and equation 4.10 is used to derive the new economic values of backfat with -0.15, +0.30, and -0.30 changes.

In order to use equation 3.51(a) in deriving a new economic value for backfat (of a +0.15 change), information of the optimal feasible solution is used. The needed information is shown in appendix B, table B.2.

Substituting relevant information into equation 3.51(a), the following new economic value for backfat is found:

For a +0.15 change in backfat

$$3.51(a) \qquad \text{E.V.} = \left[\frac{1}{589.875}\right] [\$-456.46] = \$-.77$$

In order to use equation 4.10 in deriving new economic values for backfat (of -0.15, +0.30, and -0.30 changes), new linear programs must be developed and solved. Relevant information from the optimal solutions of the new linear programs reflecting -0.15, +0.30, and -0.30 changes in backfat is shown in appendix B, table B.3. Table B.3 shows the value of the objective function for the initial and "new" linear program solutions and also the number of animals produced by the farm firm with the h-th trait improved.

Substituting the relevant information into equation 4.10, the following new economic values for backfat are found:

For a -0.15 change in backfat

$$4.10 \qquad \text{E.V.} = \left[\frac{2}{589.875 + 582.231}\right] [\$12,719.170 - \$12,242.198] = \$.81$$

If equation 3.51(a) had inappropriately been used to find the economic value of a -0.15 change in backfat, the economic value would have been $.78.

For a +0.30 change in backfat

$$4.10 \qquad \text{E.V.} = \left[\frac{2}{589.875 + 582.231}\right] [\$12,719.170 - \$12,242.198]$$

$$= \$-1.67$$

If equation 3.51(a) had inappropriately been used to find the economic value of a +0.30 change in backfat, the economic value would have been $-1.55.

For a -0.30 change in backfat

4.10 $$\text{E.V.} = [\frac{2}{589.875 + 582.231}] [\$13,190.333 - \$12,242.198]$$
$$= \$1.62$$

If equation 3.51(a) had inappropriately been used to find the economic value of a -0.30 change in backfat, the economic value would have been $1.56.

b. <u>Feed efficiency</u> Changes in linear program coefficients that reflect +0.15, -0.15, +0.30, and -0.30 changes in feed efficiency are shown in appendix B, table B.4. The -0.15, +0.30, -0.30 changes in feed efficiency cause changes in the optimal mix of activities of the optimal feasible solution of each linear program reflecting these respective changes. The +0.15 change in feed efficiency does not cause a change in the optimal mix of activities of the optimal feasible solution of the linear program reflecting the change. As a result, equation 4.10 is used to derive the new economic values of feed efficiency with -0.15, +0.30, and -0.30 changes, and equation 3.51(a) is used to derive the new economic value of feed efficiency with a +0.15 change.

In order to use equation 3.51(a) in deriving a new economic value for feed efficiency (of a +0.15 change), information of the optimal feasible solution is used. The needed information is shown in appendix B, table B.5.

Substituting relevant information into equation 3.51(a), the following new economic value for feed efficiency is found:

For a +0.15 change in feed efficiency

3.51(a) $$\text{E.V.} = [\frac{1}{595.8333}] [\$-667.333] = \$-1.12$$

As before, new linear programs must be developed and solved in order to use equation 4.10 in deriving new economic values for feed efficiency (of -0.15, +0.30, and -0.30 changes). Information from the optimal solutions of the new linear programs reflecting respective -0.15, +0.30, and -0.30 changes in feed efficiency is shown in appendix B, table B.6. Table B.6 shows the value of the objective function for the initial and "new" linear program solutions and also the number of animals produced by the farm firm with the h-th trait improved.

Substituting relevant information into equation 4.10, the following new economic values for feed efficiency are found:

For a -0.15 change in feed efficiency

4.10 $$\text{E.V.} = [\frac{2}{595.8333 + 588.113}] [\$12,941.044 - \$12,242.198]$$

$$= \$1.18$$

Inappropriately using equation 3.51(a) in deriving the economic value of feed efficiency for a -0.15 change in feed efficiency, the economic value for feed efficiency would have been $1.13.

For a +0.30 change in feed efficiency

4.10 $$\text{E.V.} = [\frac{2}{595.8333 + 466.358}] [\$11,020.139 - \$12,242.198]$$

$$= \$-2.30$$

Inappropriately using equation 3.51(a) in deriving the economic value of feed efficiency for a +0.30 change in feed efficiency, the economic value for feed efficiency would have been $-2.25.

For a -0.30 change in feed efficiency

$$4.10 \qquad E.V. = \left[\frac{2}{595.8333 + 588.113}\right] [\$13,608.995 - \$12,242.198]$$

$$= \$2.31$$

Inappropriately using equation 3.51(a) in deriving the economic value of feed efficiency for a +0.30 change in feed efficiency, the economic value of feed efficiency would have been $2.25.

c. Average daily gain   Changes in linear program coefficients that reflect +0.15, -0.15, +0.30, and -0.30 changes in average daily gain are shown in appendix B, tables B.7a and B.7b. The +0.15, -0.15, and -0.30 changes in average daily gain do not cause changes in the optimal mix of activities of the optimal feasible solution of each linear program reflecting these respective changes. The +0.30 change in average daily gain does cause a change in the optimal mix of activities of the optimal feasible solution of the linear program reflecting the +0.30 change. As a result, equation 3.51 is used to derive the new economic values of average daily gain with +0.15, -0.15, and -0.30 changes, and equation 4.10 is used to derive the new economic value of average daily gain with a +0.30 change.

In order to use equation 3.51 in deriving new economic values for average daily gain (of +0.15, -0.15, and -0.30 changes), information of the optimal feasible solution is used. The needed information is shown in appendix B, tables B.8, B.9, and B.10.

Substituting relevant information into equation 3.51, the following new economic values for average daily gain are found:

For a +0.15 change in average daily gain

$$3.51 \qquad E.V. = [\frac{1}{595.8333}] [0 + 50.375] = \$.08$$

For a -0.15 change in average daily gain

$$3.51 \qquad E.V. = [\frac{1}{595.8333}] [0 + -56.875] = \$-.10$$

For a -0.30 change in average daily gain

$$3.51 \qquad E.V. = [\frac{1}{595.8333}] [0 + -133.25] = \$-.22$$

A new linear program was developed and solved in order to use equation 4.10 in deriving a new economic value of average daily gain (of a +0.30 change) due to the change in the optimal mix of activities. Relevant information from the optimal solution of the new linear program reflecting the +0.30 change in average daily gain is shown in appendix B, table B.11. Table B.11 shows the value of the objective function for the initial and "new" linear program solutions and also the number of animals produced by the farm firm with the h-th trait improved.

Substituting the relevant information into equation 4.10, the following new economic value for average daily gain is found:

For a +0.30 change in average daily gain

$$4.10 \qquad E.V. = [\frac{2}{595.833 + 555.455}] [\$12,073.967 - \$12,242.198]$$

$$= \$-.29$$

Using equation 3.51, inappropriately, to find the economic value of average daily gain with a +0.30 change in average daily gain would have given an economic value of $-.31.

4. Comparison of economic values

Briefly comparing the economic values derived from Model I (the initial linear program of the four farrowing swine farm with the central farrowing house) and Model II (the linear program of the two farrowing swine farm with the pasture farrowing system), it can be seen that the economic values do differ among the traits. Looking at table 5.9, the absolute values of economic values derived from Model II were generally smaller in value than those derived from Model I. In only one case was a derived economic value from Model II larger than one from Model I. This was the economic value of average daily gain for a $+2\sigma$ (+0.30) change.

Even more peculiar, though, is the fact that the economic values for average daily gain for a $+2\sigma$ change derived from Model I and Model II are of opposite sign. As with the economic value derived from Model I, one would expect a positive economic value by increasing average daily gain. This is because the animal would not take as long to gain the total pounds to market weight, thereby decreasing a certain amount of variable costs due to a shorter period of time being fed. Yet, the economic value derived from Model II for a $+2\sigma$ change is negative.

The negative economic value can be explained, though. In Model II, by increasing average daily gain by +0.30 pounds of gain per day,

Table 5.9. A comparison of economic values of backfat, feed efficiency, and average daily gain derived from Models I and II

Swine enterprise	Change	Trait: Backfat	Trait: Feed efficiency	Trait: Average daily gain
Model I	+1σ	\$-.95	\$-1.44	\$ .09
	-1σ	.96	1.44	-.21
	+2σ	-1.90	-2.88	.17
	-2σ	1.91	+2.88	-.47
Model II	+1σ	-.77	-1.12	.08
	-1σ	.81	1.19	-.10
	+2σ	-1.67	-2.31	-.29
	-2σ	1.62	2.32	-.22

certain groups of hogs reach heavier weights sooner, but in a warmer season of the year, thereby requiring more finishing area per hog. Since each swine farm in the models has limited finishing areas, fewer numbers of market hogs can be fed and marketed, thereby decreasing profit. With a negative change in profit of the firm, also comes the negative economic value.

It is important to remember, though, this is a demonstration of how the economic values may vary because of the "nature of the enterprise." From this demonstration, it cannot be concluded that all two farrowing, pasture farrowing systems have smaller economic values for backfat, feed efficiency, and average daily gain, than four farrowing, central farrowing house systems. But, it is interesting to see that a positive change (or improvement) in a trait does not always mean that an increase in profit can be expected.

C. Descriptive Analysis

A Model III could be developed to further emphasize the fact that economic values may vary. Models I and II were developed on the basis of the swine farm firm. Yet, many farm firms are composed of more than just one enterprise (i.e., swine enterprise). Some farm firms are made up of cropping enterprises, beef cattle enterprises, and sometimes dairy cattle enterprises in addition to, for example, the swine enterprise.

Models that were developed using any combination of the enterprises mentioned in the above could certainly have different derived economic values than those derived from Model I, Model I with revised RHS values, or Model II. Economic values derived from the different models could be different because of the fact that inputs freed from use by the swine enterprise as a result of improving a trait of swine may be utilized to generate greater returns in one of the other farm firm enterprises. With greater returns being generated in another enterprise of the farm firm, as well as in the swine enterprise, a greater change in the profit of the farm firm is realized, resulting in a greater economic value of the trait. An example of such a case would be when feed efficiency is improved in swine such that less feed is used by the swine enterprise, but where this same feed is used in a cattle enterprise of the farm firm in order to generate greater returns, assuming a fixed level of feed available.

Economic values derived from different models could also be different because inputs of the total farm firm may generate greater returns in the swine enterprise than in any other enterprise due to a change in a swine trait. As before, with greater returns being generated by the farm firm, a greater change in the profit of the farm firm is realized, resulting in a greater economic value of the trait. An example of such a case would be when average daily gain is improved in swine such that an increased number of hogs can be fed for market (due to the decrease in space requirement per market hog). Assuming the swine enterprise to generate greater returns per dollar of total cost, and with an increased number of hogs being fed for market and a fixed level of feed available, a greater amount of feed input is needed by the swine enterprise and is given up by the cattle enterprise for use in the swine enterprise so as to generate greater returns.

D. Summary

The objective of this chapter was to demonstrate that the statement, "... Economic values of traits may vary with the particular locality or nature of the enterprise ...." [Hazel, 15, p. 487], was, in fact, true. RHS values of Model I were changed so as to represent a swine firm of a different locality with possibly fewer working hours available. Certain economic values derived from the revised Model I were different than economic values derived by Model I, as shown by table 5.6. Model II was developed so as to represent a firm with a different "nature of enterprise." Most of the economic values derived from Model II were different than economic values derived from Model I, as shown by table 5.9.

The descriptive analysis section of the chapter described how other models could be built so as to derive still more economic values that could be different in value. These models would show the change in the profit of total firm (including all enterprises) and not the change in the profit of the firm due to the change in the profit of a farm firm with a single enterprise. Both types of models will give the change in profit of the firm due to the change in the trait of each animal.

VI. THE DERIVATION PROCESS OF ECONOMIC VALUES USING AN ECONOMIC MODEL - WEAKNESSES, STRENGTHS, AND EXTENSIONS

The process of deriving economic values using an economic model was shown in chapter III. Chapters IV and V illustrated the process of deriving economic values. Weaknesses, strengths, and extensions of the procedure discussed in chapters III, IV, and V are discussed in this chapter.

A. Weaknesses

One of the major weaknesses of the process lies in the complexity of the process of forming linear program coefficients.

The most difficult coefficients to find values for were the c_j coefficients. For many activities, the c_j coefficients to be formed by using two equations (one to first find the q_{kj} value and one to then find the c_j value) and all activities required the c_j value to be discounted to present value. Many activities required up to ten variable inputs, thus causing the process of finding q_{kj} values and the discounted c_j values to be lengthy.

Certain "tricks" are available in linear programming, though, to make the formation of the economic model easier [Beneke and Winterboer, 2, pp. 53-54]. Just as gilts and boars were purchased through purchasing activities of the linear program, variable inputs (e.g., corn, soybean oilmeal, dicalcium phosphate, etc.) can be purchased through separate purchasing activities. The variable inputs are then transferred within the program by transfer rows to activities where they are utilized in the production process.

In this way, each production coefficient of variable input may be used in the same manner as the production coefficient of fixed input in the linear program tableau. An illustration of this "trick" is shown in figure 6.1.

Row description	Row name	RHS	Row type	Feed boar A10	Purchase corn A15	Purchase soybean oilmeal A16	. . .
	C-row			-5	-2.20	-.12	
Purchased corn transfer row	R15		LTE	35.8416	-1		
Purchased soybean oilmeal transfer row	R16		LTE	109.50		-1	
. . .	. . .		. . .				

Figure 6.1. Examples of transfer rows

In figure 6.1 it can be seen that there could be one purchasing activity and one transfer row for each variable input such that each production coefficient of variable feed input could be placed in the tableau as any production coefficient of fixed input. By structuring the model, as in figure 6.1, the value of C_{10} is not as tedious to find, although more values of c_j must be computed.

The value of C_{10} in figure 6.1 includes the variable costs of veterinary and medical inputs and fuel and power inputs but does not include the variable costs of purchased feed inputs. The c_j value of each feed purchasing activity is actually the cost per unit of the respective feed input (or the r_k value of equations 3.2 and 3.3).

The "trick" illustrated in figure 6.1 makes the linear program larger and more difficult to solve, but it does ease the process of developing the model. This same trick also strengthens the use of the proposed economic model, as will be seen in the next section concerning strengths of the derivation process.

A second weakness of the derivation process of economic values using an economic model lies in the inability to accurately develop the economic model so as to reflect changing marginal products. The last additional unit of input needed in producing the last unit of output rarely remains constant as production increases. There are cases where the amount of input needed to produce another unit of output continually increases. This is shown in figure 6.2.

There are also cases where the amount of input needed to produce another unit of output continually decreases. This is shown in figure 6.3.

There are methods in which the changing marginal products may partially be reflected in a linear programming procedure. By assuming a constant marginal product for respective levels of production of each production activity (which is shown by linear segments OA, OB, and OC in figure 6.2), the decreasing marginal product may partially be reflected in appropriate linear programs. This was actually done in the linear programs presented. As swine are fed to heavier weights, increased feed inputs are needed per pound of gain (i.e., feed efficiency declines as can be seen in table 4.1).

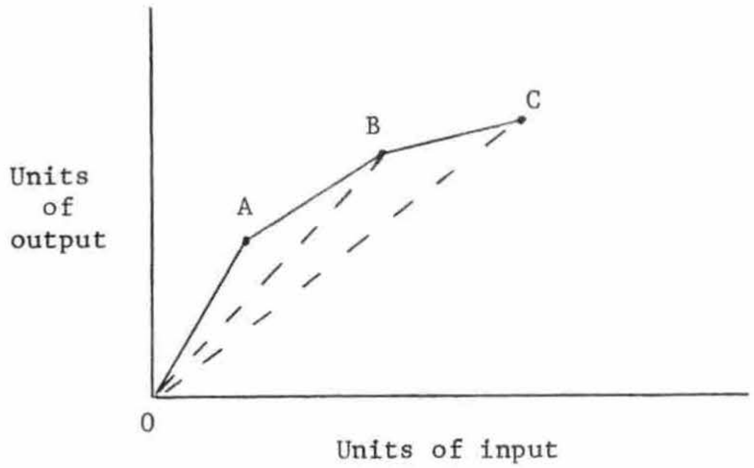

Figure 6.2. Production relation showing decreasing marginal product

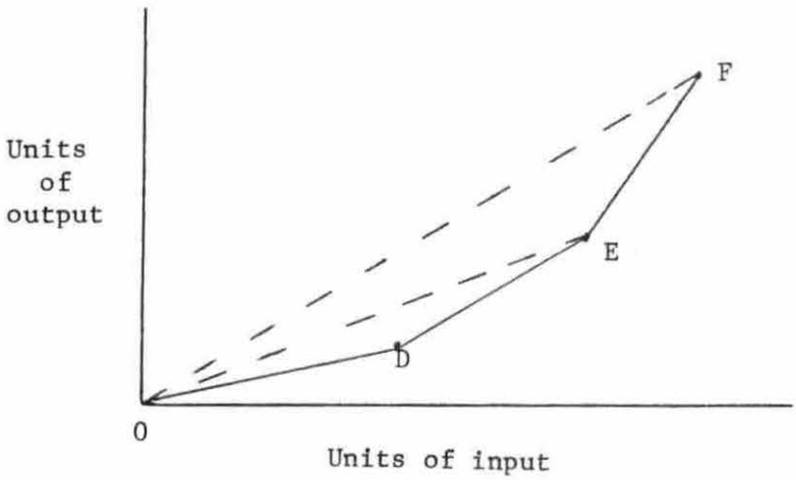

Figure 6.3. Production relation showing increasing marginal product

By assuming a different constant marginal product (i.e., a different activity) in feeding hogs fed for market to different market weights, decreasing marginal productivity was partially reflected for feeding hogs fed for market.

By assuming a constant marginal product for various levels of production of each production activity (shown by linear segments OD, OE, and OF in figure 6.2) and also by making appropriate changes in RHS values so as to solve optimal solutions for each constant marginal product and corresponding RHS value change, the increasing marginal product may partially be reflected in linear programs [Heady and Candler, 18, pp. 220-225].

The fact that changing marginal products are only partially reflected is actually not too serious. This is because the other methods of deriving the economic values, which were alluded to earlier, also assume constant marginal products. But these methods violate changing marginal products even more by assuming constant marginal products for one range of production and not assuming different constant marginal products for different respective ranges of production. Such things as the amount of labor needed per market swine are assumed to be known and constant, evading the fact that marginal products vary with the level of production.

B. Strengths

By discussing the weaknesses of the derivation process proposed by this thesis, certain strengths of the process become apparent. Earlier, in the previous section, a trick was illustrated such that the formation of c_j values did not entail as many calculations. This trick allowed the formation of purchasing activities for feed inputs and thus, the price per unit of the feed inputs to be used as c_j values so as to eliminate the calculation of variable feed costs in finding respective c_j values.

It is quite obvious that with a change in variable input prices or output prices, the economic value of certain traits may vary. Since price fluctuations have become even more prevalent in agriculture today, economic values are varying. By deriving economic values with the economic model, price changes can be dealt with quite handily. The trick, described earlier, makes dealing with price changes easier.

Any time a price change in an input or output becomes necessary, the price change can be made in the economic model and a new economic value can be found. Using the trick described earlier allows the price change of a certain input to be handled by changing the corresponding c_j coefficient of the purchasing activity of that certain input. In certain cases the economic model may be developed so as to derive economic values of traits under several price assumptions for inputs and outputs.

In the previous section, also, was mentioned the fact that certain methods of deriving economic values assumed constant marginal products for one range of production. It also follows that these certain other methods assume only one production process in deriving economic values of traits. Hazel [16], in deriving economic values for various traits, indicated costs and returns of one animal under one process of production. Hazel gave no indications that other processes may be possible or more profitable in finishing or marketing the animal.

A major strength of the proposed derivation process using an economic model is the fact that the economic model is able to include more than one possible production process in growing and finishing animals. In changing traits, the most profitable feasible production process may change. The economic model indicates this. Hazel's 1956 method assumes that the one production process is the only one that is used.

The economic model, by indicating a change in production process, also indicates the total possible profit change due to a change in the trait. Hazel [16], by indicating a profit change in the one production process, may underestimate the total possible profit change due to a change in the trait. Also, by working with a single animal, Hazel's 1956 examples may underestimate or overestimate the true economic value in that greater or fewer numbers of animals may be able to be produced as a result of a genetic change. This will be shown by the economic model when the optimal mix of activities changes.

Finally, the fundamental concepts of fixed costs and their compatibility with linear programming must be discussed. Fixed costs per animal are not constant and cannot be assumed so in deriving economic values of traits of animals. Total fixed costs of a farm remain constant in the short run, independent of the level of production. Therefore, fixed costs per animal are totally dependent upon the level of production and therefore vary with the level of production.

The economic model, by optimizing the total short-run farm, need not work with fixed costs. The fixed costs of the farm remain constant, independent of the level of production and also independent of the change in the trait. When profit changes with a change in the trait, it is actually return over variable costs that changes and not return over total costs. This was alluded to earlier by equation 3.33. Thus, by not including fixed costs in the economic model and solving for the level of production, total fixed costs and fixed costs per animal need not be assumed or used in the derivation of economic values.

All methods that include fixed costs per animal in deriving economic values on the basis of one animal, bias the derived economic values. This is because, as indicated above, fixed costs per animal may vary, due to a change in the level of production of the farm due to a change in a trait. Of course, fixed costs per animal remain constant if the level of production of the farm would be sure to remain constant. The proposed economic model uses the fact that total fixed costs of a swine farm remain constant. This is a third point of strength of the proposed economic model.

C. Extensions

Until now the thesis has been concerned with the derivation process of economic values using an economic model. Yet, the economic model can be used for other related purposes also. One purpose for which the economic model can be used, which does not require any alteration from the derivation process, is finding premiums that could be paid for breeding animals with greater breeding potential.

1. Premiums for breeding animals

The premium for breeding animals with greater breeding potential is actually found in the process of deriving economic values using certain economic models. This premium has never been discussed in the previous illustrative derivations of economic values, but will be discussed now.

The change in profit for a unit change in the h-th trait was shown in section III.F as

$$dZ/dt_h = -\sum_{i,j=1}^{m,n} \partial Z/\partial a_{io}\ \partial Z/\partial c_j\ da_{ij}/dt_h + \sum_{j=1}^{n} \partial Z/\partial c_j\ dc_j/dt_h$$

or alternatively written as

$$3.50(a) \qquad dZ/dt_h = -\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\ x_{jo}\ da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\ dc_j/dt_h$$

Now, the parents of the offspring initiate the change in the h-th trait which causes the change in profit. Therefore, in farms that market only offspring of breeding animals owned by the farm, the change in profit for a unit change in the h-th trait is the premium that may be paid in purchasing breeding animals that will cause a unit change in the h-th trait of offspring. In farms that market both offspring of breeding animals owned by the farm and offspring purchased from other farms, the change in profit for a unit change in the h-th trait of offspring of only breeding animals owned by the farm is the premium that may be paid in purchasing breeding animals that will cause a unit change in the h-th trait.

In those cases where the change in the h-th trait is so large that the computable form (equation 3.50(a)) cannot be used, the difference in the maximum values of the objective functions of the initial linear program and the linear program that reflects the change in the h-th trait change will be the premium of the breeding animals. This is shown as

6.1 $$dZ/dt_h = Z' - Z_o$$

where dZ/dt_h is defined as earlier

Z' is the value of the objective function of the optimal feasible solution of the linear program that reflects a change in the h-th trait

Z is the value of the objective function of the optimal feasible solution of the initial linear program

As the computable form was revised before so as to find the economic value of the h-th trait for each animal, the computable form can be revised so as to find the premium for each breeding animal that initiates the change in the h-th trait. The computable form for premiums for each breeding animal is shown as

6.2 $$PBA = \left[\frac{1}{\sum_{j^{*'}} x_{j^{*'}}}\right] \left[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} \right.$$

$$\left. + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \right]$$

where PBA is the premium for each breeding animal that causes change in the h-th trait

$\sum_{j*'} x_{j*'}$ is the number of breeding animals that cause change in the h-th trait

Thus, it also follows that

$$6.3 \qquad PBA = [\frac{1}{\sum_{j*'} x_{j*'}}] [Z' - Z_o]$$

for those cases where the change in the h-th trait changes the optimal mix of activities.

2. Illustrative process of deriving premiums for breeding animals

Using the empirical models presented earlier, the derivation of the premiums that could be paid for breeding animals can easily be presented. So as not to be repetitive, only one of the empirical models will be used to illustrate the derivation of premiums. The empirical model to be used in the illustrative derivation process will be Model II (or the swine farm with a pasture farrowing system that farrows twice) discussed earlier in section V.B.1.

Before the derivation process begins, though, the linear program must be revised slightly. This is because, if you remember, the Model II linear program allowed the swine farm to purchase feeder pigs. The change in profit due to a change in the h-th trait of the purchased feeder pigs is not due to the greater breeding potential of the breeding animals of the swine farm. Looking at appendix A, it can be seen that activities A40 and A41 represent the purchasing of feeder pigs. Thus, by eliminating activities A40 and A41 from the linear program, the derivation of premiums of each breeding animal can be made.

a. Optimal solution   Upon eliminating activities A40 and A41, a new optimal feasible solution must be found. The new optimal solution, of the revised Model II, is shown in tables 6.1a, 6.1b, and 6.1c.

b. Sensitivity analysis   As with the derivation of economic values, once the optimal feasible solution of the linear program is found, the premium of each breeding animal is ready to be found. As with the derivation of previous economic values, the changes in the linear program coefficients that reflect the expected change in the h-th trait of offspring of the breeding animals of greater breeding potential must be found. The procedure to follow in finding changes in linear program coefficients that reflect the change in the h-th trait, though, was demonstrated in sections IV.C.1, IV.C.2, and IV.C.3 and therefore will not be shown.

Premiums of breeding animals will be found in the following sections using the computable form for premiums for each breeding animal and, in some cases, equation 6.3. The changes of the linear program coefficients that reflect the expected change in the h-th trait will not be presented since the expected change in the h-th trait will be the same as those presented in sections V.B.3.a, V.B.3.b, and V.B.3.c.

(1) Backfat   Assume that changes of +0.15 and -0.15 in backfat are possible in offspring by purchasing breeding animals with lesser and greater breeding potential, respectively. What are the premiums that could be paid in order to purchase the breeding animals?

Table 6.1a. Optimal mix of real activities and their shadow prices: revised Model II

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty (z_j-c_j)
Purchase gilts to farrow in April	A01	--	-37.45
Purchase gilts to farrow in October	A02	--	-63.11
Purchase boar to service females	A03	1.0 boar	--
Prepare purchased gilts for breeding and farrowing in April	A04	--	--
Prepare purchased gilts for breeding and farrowing in October	A05	--	--
Feed boar	A06	1.0 boar	--
Raise gilts to farrow in April	A07	26.3158 gilts	--
Raise gilts to farrow in October	A08	1.8158 gilts	--
Prepare breeding herd for breeding and farrowing in April	A09	26.3158 gilts	--
Prepare breeding herd for breeding and farrowing in October	A10	26.3158 gilts	--
Farrowing in April	A11	25.000 gilts	--
Farrowing in October	A12	25.000 sows	--
Feed weaned April pigs to 40 pounds	A13	180.000 pigs	--
Feed weaned October pigs to 40 pounds	A14	197.5000 pigs	--
Feed 40 pound pigs farrowed in April to 180 pounds	A15	--	-4.741

Table 6.1a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty $(z_j - c_j)$
Feed 40 pound pigs farrowed in April to 200 pounds	A16	85.5503 pigs	--
Feed 40 pound pigs farrowed in April to 220 pounds	A17	--	-0.8559
Feed 40 pound pigs farrowed in April to 240 pounds	A18	--	-3.6740
Feed 40 pound pigs farrowed in April to 260 pounds	A19	92.6497 pigs	--
Feed 40 pound pigs farrowed in October to 180 pounds	A20	--	-0.019
Feed 40 pound pigs farrowed in October to 200 pounds	A21	195.525 pigs	--
Feed 40 pound pigs farrowed in October to 220 pounds	A22	--	-3.0909
Feed 40 pound pigs farrowed in October to 240 pounds	A23	--	-5.3678
Feed 40 pound pigs farrowed in October to 260 pounds	A24	--	-6.4955
Market April farrowed 180 pound market hogs	A25	--	--
Market April farrowed 200 pound market hogs	A26	169.3897 cwt.	--
Market April farrowed 220 pound market hogs	A27	--	--
Market April farrowed 240 pound market hogs	A28	--	--

Table 6.1a. Continued

Activity	Activity number A_j	Amount to be purchased, produced, or marketed x_{jo}	Income penalty $(z_j - c_j)$
Market April farrowed 260 pound market hogs	A29	238.4806 cwt.	--
Market October farrowed 180 pound market hogs	A30	--	--
Market October farrowed 200 pound market hogs	A31	387.1395 cwt.	--
Market October farrowed 220 pound market hogs	A32	--	--
Market October farrowed 240 pound market hogs	A33	--	--
Market October farrowed 260 pound market hogs	A34	--	--
Market non-conceived gilts in December	A35	3.2895 cwt.	--
Market non-conceived gilts in June	A36	3.2895 cwt.	--
Market gilts culled after first farrowing (April)	A37	0.825 cwt.	--
Market sows after November farrowing	A38	100.000 cwt.	--
Market boar in October 1973	A39	4.000 cwt.	--

Table 6.1b. Income over variable costs, Z_o: revised Model II

	Amount
Income	\$10,322.99

Table 6.1c. Fixed input use and each fixed input's shadow price: revised Model II

Fixed input	Row name	Row (constraint) number i	Amount available a_{io}	Amount used $(a_{io} - x_{n+i})$	Marginal value product $(z_{n+i} - c_{n+i})$
October 1972 labor	M01	1	160	12.062	--
November 1972 labor	M02	2	160	11.699	--
December 1972 labor	M03	3	196	37.534	--
January 1973 labor	M04	4	216	28.62	--
February 1973 labor	M05	5	192	24.87	--
March 1973 labor	M06	6	198	93.534	--
April 1973 labor	M07	7	160	142.151	--
May 1973 labor	M08	8	160	74.469	--
June 1973 labor	M09	9	160	72.51	--
July 1973 labor	M10	10	216	53.068	--
August 1973 labor	M11	11	208	51.600	--
September 1973 labor	M12	12	168	117.409	--
October 1973 labor	M13	13	160	160.000	9.983
November 1973 labor	M14	14	160	74.100	--
December 1973 labor	M15	15	196	39.105	--
January 1974 labor	M16	16	216	27.374	--
February 1974 labor	M17	17	192	29.329	--
March 1974 labor	M18	18	198	27.178	--
April 1974 labor	M19	19	160	--	--
April 1973 farrowing capacity	F01	20	25	25.000	225.379
October 1973 farrowing capacity	F02	21	25	25.000	134.082
Finishing capacity	R01	22	3250	2231.050	--
Finishing capacity	R02	23	3250	2248.538	--
Boar equality	R19	24	1	1.000	-260.728

Changes of linear program coefficients due to +0.15 and -0.15 changes in backfat are shown in appendix B, table B.1. In deriving the premiums, though, the +0.15 change in backfat causes a change in the optimal mix of activities of the optimal feasible solution, while the -0.15 change in backfat does not. As a result, equation 6.3 is used to derive the premiums for breeding animals of lesser breeding potential (a +0.15 change in backfat) and equation 6.2 is used to derive the premiums for breeding animals of greater breeding potential (a -0.15 change in backfat).

In order to use equation 6.2 in deriving premiums for breeding animals of greater breeding potential, information of the optimal feasible solution is used. The needed information is shown in appendix C, table C.1.

Substituting relevant information into equation 6.2, the following premiums for breeding animals of greater breeding potential are found:

If there are 27.6316 females responsible for the -0.15 change change in backfat of each offspring

6.2 $$PBA = \left[\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}\right] \left[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \right]$$

$$= \left[\frac{1}{27.6316}\right] [0 + \$328.547]$$

$$= \$11.89/\text{female}$$

If there is 1 boar responsible for the -0.15 change in backfat of each offspring

$$6.2 \qquad PBA = \left[\frac{1}{\sum_{j*'} x_{j*'}}\right] \left[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \right]$$

$$= \left[\frac{1}{1}\right] [0 + \$328.547]$$

$$= \$328.55/\text{boar}$$

If there are 27.6316 females and 1 boar responsible for the -0.15 change in backfat of each offspring, one-half of the change in profit is due to the females and one-half is due to the boar such that a \$5.95 premium per female is appropriate for the females and a \$163.73 premium in purchasing the boar is appropriate.[1]

In order to use equation 6.3 in deriving the premium for the breeding animals, a new linear program must be developed and solved. Relevant information from the optimal solution of the new linear program reflecting a +0.15 change in backfat is shown in appendix C, table C.2. Table C.2 shows the value of the objective function for the initial and "new" linear program solutions and also the number of breeding animals that cause change in backfat.

Substituting the relevant information into equation 6.3, the following premiums for breeding animals of lesser breeding potential are found:

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[1]This may be true only with the additional assumption that the selection differential of the boar and the females is the same. In other words, one-half of the genetic change in each offspring is due to the female and one-half of the genetic change in each offspring is due to the boar. This assumption will be made throughout the remaining thesis.

If there are 27.6316 females responsible for the +0.15 change in backfat of each offspring

$$6.3 \qquad PBA = \left[\frac{1}{\sum_{j*'} x_{j*'}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{27.6316}\right] [\$10,010.388 - \$10,322.992]$$

$$= \$-11.31/\text{female}$$

Inappropriately using equation 6.2 to derive the premiums for females of lesser breeding potential used for breeding, the premium per female would be $-11.87.

If there is 1 boar responsible for the +0.15 change in backfat of each offspring

$$6.3 \qquad PBA = \left[\frac{1}{\sum_{j*'} x_{j*'}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{1}\right] [\$10,010.388 - \$10,322.992]$$

$$= \$-312.60/\text{boar}$$

Inappropriately using equation 6.2 to derive the premium for the boar of lesser breeding potential used for breeding, the premium for the boar would be $-327.86.

If there are 27.6316 females and 1 boar responsible for the +0.15 change in backfat of each offspring, one-half of the change in profit is due to the females and one-half is due to the boar such that a $-5.66 premium per female is appropriate for the females and a $-156.30 premium in purchasing the boar is appropriate. But, if equation 6.2 was inappropriately used to derive the premiums for the females and the boar, if both caused the +0.15 change in backfat of each offspring, the premium per female would be \$-5.93 and the premium for the boar would be \$-163.93.

(2) Feed efficiency Assuming that changes of +0.15 and -0.15 in feed efficiency are possible in offspring by purchasing breeding animals with lesser and greater breeding potential, respectively. What are the premiums that could be paid in order to purchase the breeding animals?

Changes of linear program coefficients due to +0.15 and -0.15 changes in feed efficiency are shown in appendix B, table B.4. In deriving the premiums, the +0.15 change in feed efficiency causes a change in the optimal mix of activities of the optimal feasible solution. The -0.15 change in feed efficiency, though, does not cause a change in the optimal mix of activities of the optimal feasible solution. As a result, equation 6.3 is used to derive the premiums for breeding animals of lesser breeding potential (cause a +0.15 change in backfat) and equation 6.2 is used to derive premiums for breeding animals of greater breeding potential (cause a -0.15 change in feed efficiency).

As before, information of the optimal feasible solution is used in equation 6.2 in deriving premiums for breeding animals of greater breeding potential. The needed information is shown in appendix C, table C.3.

Substituting relevant information into equation 6.2, the following premiums for breeding animals of greater breeding potential are found:

If there are 27.6316 females responsible for the -0.15 change in feed efficiency of each offspring

$$6.2 \qquad PBA = \left[\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}\right] \left[-\sum_{i,j}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h\right]$$

$$= [27.6316]\ [0 + 492.351]$$

$$= \$17.82/\text{female}$$

If there is 1 boar responsible for the -0.15 change in feed efficiency of each offspring

$$6.2 \qquad PBA = \left[\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}\right] \left[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h\right]$$

$$= \left[\frac{1}{1}\right]\ [0 + 492.351]$$

$$= \$492.35/\text{boar}$$

If there are 27.6316 females and 1 boar responsible for the -0.15 change in feed efficiency of each offspring, one-half of the change in profit is due to the females and one-half of the change is due to the boar such that an \$8.91 premium per female is appropriate and a \$246.18 premium in purchasing the boar is appropriate.

In order to use equation 6.3 in deriving the premiums for the breeding animals, another linear program must be developed and solved.

Relevant information from the optimal solution of the new linear program reflecting a +0.15 change in feed efficiency is shown in appendix C, table C.4. Table C.4 shows the value of the objective function for the initial and "new" linear program solutions and also the number of breeding animals that cause the change in feed efficiency.

Substituting the relevant information into equation 6.3, the following premiums for breeding animals of lesser breeding potential are found:

If there are 27.6316 females responsible for the +0.15 change in feed efficiency of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j^{*'}} x_{j^{*'}}}] [Z' - Z_o]$$

$$= [\frac{1}{27.6316}] [\$9.857.178 - \$10,322.992]$$

$$= \$-16.86/\text{female}$$

Inappropriately using equation 6.2 to derive premiums for females of lesser breeding potential used for breeding, the premium per female would be $-17.78.

Assuming there is 1 boar responsible for the +0.15 change in feed efficiency of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j^{*'}} x_{j^{*'}}}] [Z' - Z_o]$$

$$= [\frac{1}{1}] [\$9,857.178 - \$10,322.992]$$

$$= \$-465.81/\text{boar}$$

Inappropriately using equation 6.2 to derive the premium for the boar of lesser breeding potential used for breeding, the premium for the boar would be \$-491.42.

If there are 27.6316 females and 1 boar responsible for the +0.15 change in feed efficiency of each offspring, one-half of the change in profit is due to the females and one-half of the change in profit is due to the boar such that a \$-8.43 premium per female is appropriate for the females and a \$-232.91 premium in purchasing the boar is appropriate. But, if equation 6.2 was inappropriately used to derive the premiums for the females and the boar, if both caused the +0.15 change in feed efficiency of each offspring, the premium per female would be \$-8.89 and the premium for the boar would be \$-245.71.

(3) Average daily gain Assume that changes of +0.15 and -0.15 in average daily gain are possible in offspring by purchasing breeding animals with greater and lesser breeding potential, respectively. What are the premiums that could be paid in order to purchase the breeding animals?

Changes of linear program coefficients due to +0.15 and -0.15 changes in average daily gain are shown in appendix B, table B.7a. In deriving the premiums, the +0.15 and the -0.15 changes in average daily gain cause changes in the optimal mix of activities of the optimal feasible solution. As a result, equation 6.3 is used to derive the premiums for breeding animals of greater and of lesser breeding potential (cause +0.15 and -0.15 changes in average daily gain, respectively).

In order to use equation 6.3 in deriving the premiums for the breeding animals, additional linear programs must be developed and solved. Relevant information from the optimal solutions of new linear programs reflecting +0.15 and -0.15 changes in average daily gain is shown in appendix C, table C.5. Table C.5 shows the value of the objective function for the initial and "new" linear program solutions and also the number of breeding animals that cause the change in the trait, average daily gain.

Substituting the relevant information into equation 6.3, the following premiums for breeding animals of greater breeding potential are found:

If there are 27.6316 females responsible for the +0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = \left[\frac{1}{\sum_{j*'} x_{j*'}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{27.6316}\right] [\$10{,}406.336 - \$10{,}322.992]$$

$$= \$3.02/\text{female}$$

Inappropriately using equation 6.2 to derive premiums for females of greater breeding potential used in breeding, the premiums per female would be $1.25.

If there is 1 boar responsible for the +0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}] [Z' - Z_o]$$

$$= [\frac{1}{1}] [\$10,406.336 - \$10,322.992]$$

$$= \$83.34/\text{boar}$$

Inappropriately using equation 6.2 to derive premiums for the boar of greater breeding potential used in breeding, the premium for the boar would be $34.56.

If there are 27.6316 females and 1 boar responsible for the +0.15 change in average daily gain of each offspring, one-half of the change in profit is due to the females and one-half of the change is due to the boar such that a $1.51 premium per female is appropriate for females and a $41.67 premium in purchasing the boar is appropriate. But, if equation 6.2 was inappropriately used to derive the premiums for the females and the boar, if both caused the +0.15 change in average daily gain of each offspring, the premium per female would be $.63 and the premium for the boar would be $17.28.

Substituting relevant information into equation 6.3, the following premiums for breeding animals of lesser breeding potential are found:

If there are 27.6316 females responsible for the -0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}] [Z' - Z_o]$$

$$= [\frac{1}{27.6316}] [\$10,242.220 - \$10,322.992]$$

$= \$-2.92/\text{female}$

Inappropriately using equation 6.2 to derive premiums for females of lesser breeding potential used in breeding, the premiums per female would be $-1.39.

If there is 1 boar responsible for the -0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = \left[\frac{1}{\sum_{j*'} x_{j*'}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{1}\right] [\$10{,}242.220 - \$10{,}322.992]$$

$$= \$-80.77/\text{boar}$$

Inappropriately using equation 6.2 to derive premiums for the boar of lesser breeding potential used in breeding, the premium for the boar would be $-38.37.

If there are 27.6316 females and 1 boar responsible for the -0.15 change in average daily gain of each offspring, one-half of the change in profit is due to the females and one-half is due to the boar such that a $-1.46 premium per female is approprirate for females and a $-40.39 premium is appropriate in purchasing the boar. But, if equation 6.2 was inappropriately used to derive the premiums for the females and the boar, assuming both contributed to the -0.15 change in average daily gain of each offspring, the premium per female would be $-0.70 and the premium for the boar would be $-19.19.

<u>(4) Backfat, feed efficiency, and average daily gain</u> In practical animal breeding, it is naive to assume that breeding animals can be purchased so as to change one specific trait in offspring. Many traits are correlated such that if one trait is changed, other traits will be indirectly changed. This was alluded to earlier in section II.

The economic models which have been presented, demonstrating the derivation of economic values of traits and the derivation of premiums given to breeding animals of greater or lesser breeding potential, need not follow the naive assumption of changing one specific trait at a time. Although this assumption is necessary in deriving economic values of traits, it still need not be followed in deriving premiums of breeding animals.

Assume that the computable form is again given as

$$3.50(a) \qquad dZ/dt_h = -\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h$$

Now, when several implicit variables t_h are changed simultaneously, it is known that

$$6.4 \qquad dZ = \sum_h (dZ/dt_h)\, dt_h$$

such that by substitution

$$6.5 \qquad dZ = \sum_h \Big[-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \Big]\, dt_h$$

Thus, it is known that

$$6.2(a) \qquad PBA = \sum_h \Big[\Big(\frac{1}{\sum_{j^{*'}} x_{j^{*'}}}\Big) \Big(-\sum_{i,j=1}^{m,n} (z_{n+i} - c_{n+i})\, x_{jo}\, da_{ij}/dt_h + \sum_{j=1}^{n} x_{jo}\, dc_j/dt_h \Big) \Big]$$

for premiums of breeding animals with greater or lesser breeding potential due to changing more than one trait in the offspring.

This computable form, as all others presented in this thesis, though, can be used to derive premiums only if the optimal mix of activities of the optimal feasible solution remains the same. In those cases where the optimal mix of activities of the optimal feasible solution changes due to changing more than one trait, equation 6.3 can be used to derive the premium of the breeding animals as it was used in deriving the premium of the breeding animals for changes in one trait in the offspring.

Assume that a change of +0.15 in backfat and feed efficiency and a change of -0.15 in average daily gain occur simultaneously. Also assume that a change of -0.15 in backfat and feed efficiency and a change of +0.15 in average daily gain occur simultaneously. Finally, assume that the first group of changes is due to using breeding animals of lesser breeding potential and that the second group of changes is due to using breeding animals of greater breeding potential. What are the premiums that could be paid in order to purchase the two types of breeding animals?

Changes of linear program coefficients due to changes in backfat, feed efficiency, and average daily gain are shown in appendix B in tables B.1, B.4, and B.7a, respectively. In deriving the premiums of breeding animals of lesser breeding potential (cause +0.15 changes in backfat and feed efficiency and a -0.15 change in average daily gain), the optimal mix of activities of the optimal feasible solution changes. The same is true in deriving the premiums of breeding animals of greater breeding potential (cause -0.15 changes in backfat and feed efficiency and a +0.15 change in average daily gain). As a result, equation 6.3 is used to derive the premiums of the breeding animals.

In order to use equation 6.3 in deriving premiums for breeding animals, linear programs must be developed to reflect the changes in the traits and must then be solved. Relevant information from the optimal solutions of the "new" linear programs is shown in appendix C, table C.6. Table C.6 shows the value of the objective function for the initial and "new" linear program solutions and also the number of breeding animals that cause the changes in the traits.

Substituting the relevant information into equation 6.3, the following premiums for breeding animals of lesser breeding potential are found:

If there are 27.6316 females responsible for the +0.15 changes in backfat and feed efficiency and the -0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j*'} x_{j*'}}] [Z' - Z_o]$$

$$= [\frac{1}{27.6313}] [\$9,486.238 - \$10,322.992]$$

$$= \$-30.28/\text{female}$$

If equation 6.2(a) had been used to derive the premium for females, which would have been inappropriate, the premium per female would have been $-31.04. This figure can be found by adding the premiums of female breeding animals derived by using equation 6.2 for changes of +0.15 in backfat ($-11.87), +0.15 in feed efficiency ($-17.78), and -0.15 in average daily gain ($-1.39).

If there is 1 boar responsible for the +0.15 changes in backfat and feed efficiency and the -0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = [\frac{1}{\sum_{j*'} x_{j*'}}] [Z' - Z_o]$$

$$= [\frac{1}{1}] [\$9,486.238 - \$10,322.992]$$

$$= \$-836.75/\text{boar}$$

If equation 6.2(a) had been used to derive the premium for the boar, which would have been inappropriate, the premium for the boar would have been \$-857.65. This figure can be found by adding the premiums for the boar derived by using equation 6.2 for changes of +0.15 in backfat (\$-327.86), +0.15 in feed efficiency (\$-491.42), and -0.15 in average daily gain (\$-38.37).

If there are 27.6316 females and 1 boar responsible for the changes of +0.15 in backfat and feed efficiency and a -0.15 change in average daily gain, one-half of the change in profit due to the changes in the traits is due to the females and one-half is due to the boar such that a \$-15.14 premium per female is appropriate and a \$-418.37 premium in purchasing the boar is appropriate. But, if equation 6.2(a) was inappropriately used to derive the premiums for the females and the boar, if the changes in the traits were due to both of them, the premium per female would be \$-15.52 and the premium for the boar would be \$-428.83.

Substituting the relevant information into equation 6.3, the following premium for breeding animals of greater breeding potential are found:

If there are 27.6316 females responsible for the -0.15 changes in backfat and feed efficiency and the +0.15 change in average daily gain of each offspring

6.3 $$PBA = \left[\frac{1}{\sum_{j^{*'}} x_{j^{*'}}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{27.6316}\right] [\$11{,}248.404 - \$10{,}322.992]$$

$$= \$33.49/\text{female}$$

If equation 6.2(a) had been inappropriately used to derive the premium for females, the premium would have been \$30.96. This figure can be found by adding the premiums of female breeding animals derived by using equation 6.2 for changes of -0.15 in backfat (\$11.88/female), -0.15 in feed efficiency (\$18.82/female), and +0.15 in average daily gain (\$1.35/female).

If there is 1 boar responsible for the -0.15 changes in backfat and feed efficiency and the +0.15 change in average daily gain of each offspring

$$6.3 \qquad PBA = \left[\frac{1}{\sum_{j^{*\prime}} x_{j^{*\prime}}}\right] [Z' - Z_o]$$

$$= \left[\frac{1}{1}\right] [\$11{,}248.404 - \$10{,}322.992]$$

$$= \$925.41/\text{boar}$$

If equation 6.2(a) had been used to derive the premium for the boar, which would have been inappropriate, the premium for the boar would have been \$855.46. This figure can be found by adding premiums for the boar derived using equation 6.2 for changes of -0.15 in backfat (\$328.55), -0.15 in feed efficiency (\$492.35), and +0.15 in average daily gain (\$34.56).

If there are 27.6316 females and 1 boar responsible for the changes of -0.15 in backfat and feed efficiency and a +0.15 change in average daily gain, one-half of the change in profit due to the changes in the traits is due to the females and one-half is due to the boar such that the appropriate premium per female is \$16.75 and the appropriate premium for the boar is \$462.71. If equation 6.2(a) had been inappropriately used instead of equation 6.3 to derive the premiums for the females and the boar, assuming the changes in the traits of offspring were due to both of them, the premium per female would have been \$15.48 and the premium for the boar would have been \$427.73.

D. Summary

Weaknesses, strengths, and extensions of the process of deriving economic values using economic models were discussed in this chapter. One of the major weaknesses of the derivation process of economic values using economic models lies in the complexity of the process of forming linear program coefficients. Many computations are carried on in formulating coefficients. A "trick" was shown so as to eliminate some of the computations that must be done in formulating c_j coefficients. A second weakness mentioned concerning the economic model is the inability for the model to accurately reflect changing marginal products. This, though, is also prevalent in other methods of deriving economic values.

One of the major strengths of the economic model is the ability to handle price changes. In certain cases, by changing one coefficient of the linear program, the effect of a change in price of an input or output may be determined. Another strength of the economic model is the greater number of production processes available in deriving economic values. A change in the h-th trait may also justify a possible change in the production process. The economic model indicates this; other methods of deriving economic values have not. A third strength of the economic model is the manner in which fixed costs are handled.

The third section of the chapter was devoted to discussing the derivation of breeding animal premiums. As with the process of deriving economic values, a computable form may be used to derive premiums, provided the optimal mix of activities does not change. In the case where there is a change in the optimal mix of activities, an alternative procedure is available in which the premiums may be derived. Premiums can be derived for only breeding females, only breeding males, and for both breeding females and breeding males if both female and male contribute to the change in the trait. Premiums can also be derived for breeding animals when there is a change in more than one trait of each offspring of the breeding animals.

VII. SUMMARY

Basic concepts of genetics and animal breeding were presented, early in the thesis, so that some of the relationships that exist between the specialized fields of genetics, animal breeding, and economics could be shown. The selection index was presented as a tool which uses basic concepts of genetics, animal breeding, and economics to simultaneously select for several traits in an effort to make maximum genetic improvement.

Underlying principles of the selection index were discussed so that a clearer understanding of the selection index could be obtained by the reader. It was seen that certain parameters of a livestock population must be known before the selection index can be used. One parameter which must be known, and which was the main subject of the thesis, was the economic value (or economic weight) of each trait.

Prior to the use of an economic value of a trait as a parameter in the selection index and certainly before the economic value can be assumed to be known, the economic value of the trait must be defined and must also be capable of being found. The working definition of an economic value of a trait, though, is rather nebulous in the literature on selection indexes. One definition that is most common and seems to be acceptable among animal breeders is that an economic value of a trait is the amount by which profit may be expected to change for each unit of improvement in the trait.

Many methods of deriving economic values of traits have been proposed and used on the basis of the previous definition of economic values. After altering the cited working definition of economic values slightly, an objective of the thesis was to propose a method of deriving economic values of traits using an economic model. The altered definition of economic values on which the derivation process was based, was that an economic value of a trait is the amount by which profit of the firm may be expected to increase for each unit of improvement in a trait of each animal. The basis on which the economic model was developed was linear programming and linear programming theory of a profit maximizing firm.

As it was necessary to discuss certain fundamental concepts of genetics and animal breeding, it was also necessary to reveal fundamental concepts of linear programming. From the fundamental concepts of linear programming, it was possible to develop an economic theory of a competitive profit-maximizing firm, from which an economic model was developed.

So that the reader would more fully understand linear programming and the economic model from which the economic values of traits were derived, linear programming was presented strictly as a mathematical technique used to solve problems. The typical maximization linear program was shown. A procedure to use in solving the maximization linear program was also shown. Finally, information from the optimal feasible solution was discussed and illustrated.

The second phase of deriving economic values using the economic model was presented by introducing sensitivity analysis. Sensitivity analysis was first viewed through applications in animal breeding. A symbolic representation of sensitivity analysis was then presented.

By following the symbolic representation of sensitivity analyses, a computable form was formulated to find changes in the value of the objective function due to changes in linear program coefficients.

Further manipulations with the computable form allowed the derivation of the revised computable form which was used to derive the economic values of traits. The revised computable form, though, as the computable form, will only reveal the true economic value of the trait of each animal provided the optimal mix of activities of the optimal feasible solution of the economic model does not change with changes in the linear program coefficients reflecting the change in the trait.

Thus, from presenting the above material, it was summarized that in deriving economic values of traits using an economic model, a farm firm must first be developed. After the development of the farm firm, the farm firm must be put into a linear program problem by forming basic parameters of the linear program to reflect the farm firm and thereby developing an economic model of the farm firm. Using the simplex method, the optimal combination of inputs and output can be determined so as to maximize the farm firm's profit. Finally, by substituting into the revised computable form, information from the optimal solution of the linear program, and by changing certain parameters so as to reflect a change in a trait, the revised computable form will give the economic value of the trait that was to be found.

So that the reader would strengthen his understanding of the process of deriving economic values using an economic model, an empirical economic model of a swine farm was developed so as to derive economic values for backfat, feed efficiency, and average daily gain of swine. A general description of the swine farm was first given. Then, basic assumptions needed in developing the economic model were given. Using the basic assumptions, the formation of certain linear program coefficients was demonstrated. Finally, the specific description of the linear program of the swine farm was presented.

Following the presentation of the optimal feasible solution of the linear program of the swine farm, sensitivity analyses of the optimal feasible solution were used to find economic values for the backfat, feed efficiency, and average daily gain. The sensitivity analysis for each trait included a discussion of the linear program coefficients that reflect a change in the trait, a discussion of the amount of change in the coefficients that would be appropriate, and a discussion of the use of the revised computable form. Derived economic values were also presented in the sensitivity analyses sections.

Because the revised computable form may not be appropriate to use in all derivations of economic values of traits because of changes in the optimal mix of activities, changes in the optimal mix of activities of each optimal solution of linear programs reflecting changes in respective traits were analyzed. It was demonstrated that the revised computable form does not determine the correct economic value of a trait if a large change in a trait causes a change in the optimal mix of activities. It was also demonstrated by solving new linear programs that reflect the respective changes in traits that economic values can be derived by finding the difference between the maximum value of the objective function of the new linear program and the maximum value of the objective function of the initial linear program and dividing that difference by the average number of animals with trait changes between the two linear programs. Since the only way to determine if the optimal mix of activities does change (upon changing linear program coefficients so as to reflect a change in a trait), though, is by solving for an optimal feasible solution to a new linear program, it may be advantageous to exclude the use of the revised computable form from the derivation process. Yet, if one is sure the change in the trait is small enough so as not to change the optimal mix of activities, the revised computable form is an excellent tool to use in deriving economic values of traits.

After the illustrative analysis of deriving economic values using an economic model, it was demonstrated that economic values of traits may vary with the "particular locality" and may vary with the "nature of the enterprise." By revising RHS values of Model I so as to reflect a different locality, new economic values were derived. By following the illustrative procedures in developing Model I, Model II was developed so as to reflect a different "nature of the enterprise," and so as to derive new economic values.

All economic values of the respective traits, backfat, feed efficiency, and average daily gain, derived from Model I, revised Model I (with RHS values of Model I changed), and Model II, found by using the revised computable form and the alternative derivation formula (used when the optimal mix of activities of the optimal feasible solution changes), are shown in table 7.1. The empty places

Table 7.1. Economic values of backfat, feed efficiency, and average daily gain derived from Model I, revised Model I, and Model II

		Change in the trait							
		+1σ		-1σ		+2σ		-2σ	
Model	Trait	Computable form	Alternative formula	Computable form	Alternative formula	Computable form	Alternative formula	Computable form	Alternative formula
Model I	Backfat	$ -.95		$ .96		$-1.90		$ 1.91	
	Feed efficiency	-1.44		1.44		-2.88		2.88	
	Average daily gain	.09		-.11	$ -.21	.17		-.24	$ -.47
Revised Model I	Backfat	-.95		.96		-1.90		1.91	
	Feed efficiency	-1.44		1.44		-2.87		2.88	
	Average daily gain	1.08	$.94	-1.58	-1.02	1.89	$1.02	-1.72	-1.37
Model II	Backfat	-.77		.78	.81	-1.55	-1.67	1.56	1.62
	Feed efficiency	-1.12		1.13	1.18	-2.25	-2.30	2.25	2.31
	Average daily gain	.08		-.10		-.31	-.29	-.22	

under the alternative formula columns indicate that the alternative formula was not needed in deriving the true economic values of the respective traits since there was no change in the optimal mix of activities with change in the respective trait.

In addition to the illustrative derivations of economic values, it was described how other models could be developed so as to derive economic values for traits of swine that would be different in value due to the models' including other enterprises in addition to swine. These models would show the change in the profit of the total firm (including all enterprises) and not the change in the profit of the firm due to the change in the profit of a single enterprise. It was indicated that this profit would also be the change in profit of the firm due to the change in the trait of each animal.

Weaknesses, strengths, and extensions of the process of deriving economic values using economic models were also discussed. One of the major weaknesses discussed was the complexity of the process of forming linear program coefficients. Many computations are carried out in formulating coefficients. A "trick" was shown, though, so as to reduce the required computations needed in formulating c_j coefficients. A second weakness, mentioned concerning the economic model, was its inability to accurately reflect changing marginal products. This, though, was said to be prevalent in other methods of deriving economic values.

One of the major strengths discussed was the ability to handle price changes within the economic model. Another strength of the economic model was its ability to handle a number of production processes.

In certain cases, a change in a trait will cause a change in the production process. Those derivation methods which analyze only one production process do not indicate a possible change in the production process. A third strength of the economic model discussed was the manner in which fixed costs are handled.

Finally, an extension of the derivation of economic values using an economic model was discussed. This extension was the derivation of breeding animal premiums. So that the derivation of breeding animal premiums be thoroughly understood, the methodology and illustrative analyses were presented. As with the process of deriving economic values, a computable form was used to derive the premiums, provided the optimal mix of activities did not change. In the case where there was a change in the optimal mix of activities, an alternative procedure was available to derive the premiums.

Premiums were derived for only females, only males, and for both females and males, assuming both contribute to the change in the trait of offspring. Premiums were also derived for breeding animals assuming a change in more than one trait. The premiums are shown in table 7.2. The empty places under the alternative formula columns indicate that the alternative formula was not needed in deriving true premiums of the breeding animals, since there was no change in the optimal mix of activities with change in the respective trait.

Other extensions of the process of deriving economic values using an economic model may be made. It is difficult, though, to conceive and specify all the possible extensions without close collaboration among breeders, geneticists, and economists. Only now are animal

Table 7.2. Premiums of breeding animals derived from revised Model II, assuming changes in the traits of offspring

		Breeding animals							
						Male and female			
		Male		Female		Male		Female	
Trait	Change	Comput-able form	Alter-native formula	Comput-able form	Alter-native formula	Comput-able form	Alter-native formula	Comput-able form	Alter-native formula
Backfat	+0.15	$-327.86	$-312.60	$-11.87	$-11.31	$-163.93	$-156.30	$ -5.93	$ -5.66
	-0.15	328.55		11.89		164.27		5.95	
Feed efficiency	+0.15	-491.42	-465.81	-17.78	-16.86	-245.71	-232.91	-8.89	-8.43
	-0.15	492.35		17.82		246.18		8.91	
Average daily gain	+0.15	34.56	83.34	1.25	3.02	17.28	41.67	.63	1.51
	-0.15	-38.37	-80.77	-1.39	-2.92	-19.19	-40.39	-.70	-1.46
Backfat Feed efficiency Average daily gain	+0.15 +0.15 -0.15	-857.65	-836.75	-31.04	-30.28	-428.83	-418.37	-15.52	-15.14
Backfat Feed efficiency Average daily gain	-0.15 -0.15 +0.15	855.46	925.41	30.96	33.49	427.73	462.71	15.48	-16.75

breeders, geneticists, and economists, at Iowa State University, beginning to formally collaborate through interdisciplinary workshops. These are needed so that future work in the economics of breeding may be possible.

This thesis is a result of collaboration among animal scientists and economists. Hopefully, further collaboration will be carried out so that other theses may be initiated and further work may be done in the area of the economics of breeding and even other areas where animal science and economics mix.

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X. APPENDIX A

Row description	Row name	RHS	Row type	Purchase gilts in October A01	Purchase gilts in April A02	Purchase boar in October A03	Prepare October purchased gilts for breeding A04	Prepare April purchased gilts for breeding A05	Feed boar A06
	C-row			-104.80	-124.34	-277.78	-11.22	-9.22	-102.56
October 1972 labor	M01	160	LTE	.15		.1			.12
November 1972 labor	M02	160	LTE				.70		.12
December 1972 labor	M03	196	LTE						.12
January 1973 labor	M04	216	LTE						.12
February 1973 labor	M05	192	LTE						.12
March 1973 labor	M06	198	LTE						.12
April 1973 labor	M07	160	LTE		.15				.12
May 1973 labor	M08	160	LTE					.68	.12
June 1973 labor	M09	160	LTE						.12
July 1973 labor	M10	216	LTE						.12
August 1973 labor	M11	208	LTE						.12
September 1973 labor	M12	168	LTE						.12
October 1973 labor	M13	160	LTE						
November 1973 labor	M14	160	LTE						
December 1973 labor	M15	196	LTE						
January 1974 labor	M16	216	LTE						
February 1974 labor	M17	192	LTE						
March 1974 labor	M18	198	LTE						
April 1974 labor	M19	160	LTE						
April farrow. cap.	F01	25	LTE						
October farrow. cap.	F02	25	LTE						
Finishing capacity	R01	3250	LTE						
Finishing capacity	R02	3250	LTE						
Transfer row for gilts purchased in October	R17		LTE	-1			+1		

Figure A.1. Linear program tableau of Model II

Row description	Row name	RHS	Row type	Purchase gilts in October A01	Purchase gilts in April A02	Purchase boar in October A03	Prepare October purchased gilts for breeding A04	Prepare April purchased gilts for breeding A05	Feed boar A06
Transfer row for gilts purchased in April	R18		LTE		-1			+1	
Boar equality	R19	1	E			+1			
Transfer row for gilts prepared for breeding	R20		LTE				-.95		
Transfer row for gilts prepared for breeding	R21		LTE					-.95	
Transfer row for the purchased boar	R22		E			-1			+1
Transfer row to market boar	R23		E						-4
Transfer row for breeding herd preparation	R24		LTE						
Transfer row for breeding herd preparation	R25		LTE						
Transfer row to market October farrowed sows	R26		LTE						
Transfer row for non-breeder gilts to market	R27		LTE				-.125		

Figure A.1. Continued

Row description	Row name	RHS	Row type	Purchase gilts in October A01	Purchase gilts in April A02	Purchase boar in October A03	Prepare October purchased gilts for breeding A04	Prepare April purchased gilts for breeding A05	Feed boar A06
Transfer row for non-breeder gilts to market	R28		LTE					-.125	
Transfer row for non-breeder and culled sows to market	R29		LTE						
Weaned April pig transfer row	R30		LTE						
Weaned October pig transfer row	R31		LTE						
40 pound April pig transfer row	R32		LTE						
40 pound October pig transfer row	R33		LTE						
180 pound April pig transfer row	R34		LTE						
200 pound April pig transfer row	R35		LTE						
220 pound April pig transfer row	R36		LTE						
240 pound April pig transfer row	R37		LTE						
260 pound April pig transfer row	R38		LTE						
180 pound October pig transfer row	R39		LTE						

Figure A.1. Continued

Row description	Row name	RHS	Row type	Purchase gilts in October A01	Purchase gilts in April A02	Purchase boar in October A03	Prepare October purchased gilts for breeding A04	Prepare April purchased gilts for breeding A05	Feed boar A06
200 pound October pig transfer row	R40		LTE						
220 pound October pig transfer row	R41		LTE						
240 pound October pig transfer row	R42		LTE						
260 pound October pig transfer row	R43		LTE						

Figure A.1. Continued

Row name	Raise gilts to farrow in April A07	Raise gilts to farrow in October A08	Prepare breeding herd to farrow in April A09	Prepare breeding herd to farrow in October A10	April farrow A11	October farrow A12	Start April pigs to 40 pounds A13	Start October pigs to 40 pounds A14	Feed April pigs to 180 pounds A15	Feed April pigs to 200 pounds A16	Feed April pigs to 220 pounds A17
C-row	-66.59	-60.39	-11.98	-10.06	-52.21	-47.89	-6.94	-7.42	-27.63	-31.71	-35.92
M01	.45										
M02			.44								
M03					1.49						
M04					1.14						
M05					.99						
M06					3.73						
M07		.43			5.65						
M08				.43			.35				
M09						1.47			.2	.2	.2
M10						1.12			.14	.14	.14
M11						.99			.15	.15	.15
M12						3.66				.04	.08
M13						5.77					
M14								.36			
M15											
M16											
M17											
M18											
M19											
F01					+1						
F02						+1					
R01									+12	+12	+12
R02											
R17											

Figure A.1. Continued

Row name	Raise gilts to farrow in April A07	Raise gilts to farrow in October A08	Prepare breeding herd to farrow in April A09	Prepare breeding herd to farrow in October A10	April farrow A11	October farrow A12	Start April pigs to 40 pounds A13	Start October pigs to 40 pounds A14	Feed April pigs to 180 pounds A15	Feed April pigs to 200 pounds A16	Feed April pigs to 220 pounds A17
R18											
R19											
R20			-.95		+1						
R21				-.95		+1					
R22											
R23											
R24	-1		+1								
R25		-1		+1	-.98						
R26						-4					
R27			-.125								

Figure A.1. Continued

Row name	Raise gilts to farrow in April A07	Raise gilts to farrow in October A08	Prepare breeding herd to farrow in April A09	Prepare breeding herd to farrow in October A10	April farrow A11	October farrow A12	Start April pigs to 40 pounds A13	Start October pigs to 40 pounds A14	Feed April pigs to 180 pounds A15	Feed April pigs to 200 pounds A16	Feed April pigs to 220 pounds A17
R28				-.125							
R29					-.033						
R30					-7.2		+1				
R31						-7.9		+1			
R32							-.99		+1	+1	+1
R33								-.99			
R34									-1.782		
R35										-1.98	
R36											-2.178
R37											
R38											
R39											

Figure A.1. Continued

Row name	Raise gilts to farrow in April A07	Raise gilts to farrow in October A08	Prepare breeding herd to farrow in April A09	Prepare breeding herd to farrow in October A10	April farrow A11	October farrow A12	Start April pigs to 40 pounds A13	Start October pigs to 40 pounds A14	Feed April pigs to 180 pounds A15	Feed April pigs to 200 pounds A16	Feed April pigs to 220 pounds A17
R40											
R41											
R42											
R43											

Figure A.1. Continued

Row name	Feed April pigs to 240 pounds A18	Feed April pigs to 260 pounds A19	Feed Oct. pigs to 180 pounds A20	Feed Oct. pigs to 200 pounds A21	Feed Oct. pigs to 220 pounds A22	Feed Oct. pigs to 240 pounds A23	Feed Oct. pigs to 260 pounds A24	Market 180 pound April hogs A25	Market 200 pound April hogs A26	Market 220 pound April hogs A27	Market 240 pound April hogs A28	Market 260 pound April hogs A29
C-row	-40.34	-44.87	-26.01	-29.85	-33.82	-37.97	-42.24	40.15	40.59	38.44	36.41	36.99
M01												
M02												
M03												
M04												
M05												
M06												
M07												
M08												
M09	.2	.2										
M10	.14	.14										
M11	.15	.15										
M12	.15	.15						.05	.05	.05		
M13		.04									.05	.05
M14												
M15			.2	.2	.2	.2	.2					
M16			.14	.14	.14	.14	.14					
M17			.15	.15	.15	.15	.15					
M18				.04	.08	.15	.15					
M19							.04					
F01												
F02												
R01	+12.5	+13										
R02			+10	+11.5	+12	+12.5	+13					
R17												

Figure A.1. Continued

Row name	Feed April pigs to 240 pounds A18	Feed April pigs to 260 pounds A19	Feed Oct. pigs to 180 pounds A20	Feed Oct. pigs to 200 pounds A21	Feed Oct. pigs to 220 pounds A22	Feed Oct. pigs to 240 pounds A23	Feed Oct. pigs to 260 pounds A24	Market 180 pound April hogs A25	Market 200 pound April hogs A26	Market 220 pound April hogs A27	Market 240 pound April hogs A28	Market 260 pound April hogs A29
R18												
R19												
R20												
R21												
R22												
R23												
R24												
R25												
R26												
R27												

Figure A.1. Continued

Row name	Feed April pigs to 240 pounds A18	Feed April pigs to 260 pounds A19	Feed Oct. pigs to 180 pounds A20	Feed Oct. pigs to 200 pounds A21	Feed Oct. pigs to 220 pounds A22	Feed Oct. pigs to 240 pounds A23	Feed Oct. pigs to 260 pounds A24	Market 180 pound April hogs A25	Market 200 pound April hogs A26	Market 220 pound April hogs A27	Market 240 pound April hogs A28	Market 260 pound April hogs A29
R28												
R29												
R30												
R31												
R32	+1	+1										
R33			+1	+1	+1	+1	+1					
R34								+1				
R35									+1			
R36										+1		
R37	-2.376										+1	
R38		-2.574										+1
R39			-1.782									

Figure A.1. Continued

Row name	Feed April pigs to 240 pounds A18	Feed April pigs to 260 pounds A19	Feed Oct. pigs to 180 pounds A20	Feed Oct. pigs to 200 pounds A21	Feed Oct. pigs to 220 pounds A22	Feed Oct. pigs to 240 pounds A23	Feed Oct. pigs to 260 pounds A24	Market 180 pound April hogs A25	Market 200 pound April hogs A26	Market 220 pound April hogs A27	Market 240 pound April hogs A28	Market 260 pound April hogs A29
P40				-1.98								
P41					-2.178							
P42						-2.376						
P43							-2.574					

Figure A.1. Continued

Row name	Market 180 pound Oct. hogs A30	Market 200 pound Oct. hogs A31	Market 220 pound Oct. hogs A32	Market 240 pound Oct. hogs A33	Market 260 pound Oct. hogs A34	Market non-conceived gilts in Dec. A35	Market non-conceived gilts in June A36	Market culled gilts in May A37	Market sows in Nov. A38	Market boar in Oct. A39	Purchase feeder pigs in May A40	Purchase feeder pigs in Nov. A41
C-row	31.49	30.29	27.94	26.40	25.59	24.90	36.37	29.33	32.99	30.30	-27.83	-28.95
M01												
M02												
M03						.05						
M04												
M05												
M06												
M07												
M08								.04			.1	
M09							.05					
M10												
M11												
M12												
M13										.03		
M14									.03			.1
M15												
M16												
M17												
M18	.05	.05	.05									
M19				.05	.05							
F01												
F02												
R01												
R02												
R17												

Figure A.1. Continued

Row name	Market 180 pound Oct. hogs A30	Market 200 pound Oct. hogs A31	Market 220 pound Oct. hogs A32	Market 240 pound Oct. hogs A33	Market 260 pound Oct. hogs A34	Market non-conceived gilts in Dec. A35	Market non-conceived gilts in June A36	Market culled gilts in May A37	Market sows in Nov. A38	Market boar in Oct. A39	Purchase feeder pigs in May A40	Purchase feeder pigs in Nov. A41
R18												
R19												
R20												
R21												
R22												
R23										+1		
R24												
R25												
R26									+1			
R27						+1						

Figure A.1. Continued

Row name	Market 180 pound Oct. hogs A30	Market 200 pound Oct. hogs A31	Market 220 pound Oct. hogs A32	Market 240 pound Oct. hogs A33	Market 260 pound Oct. hogs A34	Market non-con-ceived gilts in Dec. A35	Market non-con-ceived gilts in June A36	Market culled gilts in May A37	Market sows in Nov. A38	Market boar in Oct. A39	Purchase feeder pigs in May A40	Purchase feeder pigs in Nov. A41
R28							+1					
R29								+1				
R30												
R31												
R32											-1	
R33												-1
R34												
R35												
R36												
R37												
R38												
R39	+1											

Figure A.1. Continued

Row name	Market 180 pound Oct. hogs A30	Market 200 pound Oct. hogs A31	Market 220 pound Oct. hogs A32	Market 240 pound Oct. hogs A33	Market 260 pound Oct. hogs A34	Market non-con-ceived gilts in Dec. A35	Market non-con-ceived gilts in June A36	Market culled gilts in May A37	Market sows in Nov. A38	Market boar in Oct. A39	Purchase feeder pigs in May A40	Purchase feeder pigs in Nov. A41
P40		+1										
P41			+1									
P42				+1								
P43					+1							

Figure A.1. Continued

XI. APPENDIX B

Table B.1. Changes of linear program coefficients due to +0.15, -0.15, +0.30, and -0.30 changes in backfat: Model II

j	+0.15 change in backfat dc_j/dt_h	-0.15 change in backfat dc_j/dt_h	+0.30 change in backfat dc_j/dt_h	-0.30 change in backfat dc_j/dt_h
25	$-.41	$+.43	$-.84	$+.85
26	-.43	+.42	-.85	+.85
27	-.42	+.43	-.85	+.85
28	-.42	+.42	-.84	+.85
29	-.42	+.43	-.84	+.85
30	-.39	+.41	-.79	+.80
31	-.40	+.40	-.80	+.80
32	-.40	+.40	-.80	+.80
33	-.40	+.39	-.79	+.80
34	-.40	+.40	-.79	+.80

Table B.2. Elements in equation 3.51(a) needed to find the economic value of backfat for a +0.15 change in backfat: Model II

j	x_{jo}	x_j	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
25	0	0	$-.41	0
26	536.25	268.125	-.43	$-230.59
27	0	0	-.42	0
28	0	0	-.42	0
29	0	0	-.42	0
30	579.15	321.75	-.39	-225.87
31	0	0	-.40	0
32	0	0	-.40	0
33	0	0	-.40	0
34	0	0	-.40	0
Σ		589.875		-456.46

Table B.3. Elements in equation 4.10 needed to find economic values of backfat for -0.15, +0.30, and -0.30 changes in backfat: Model II

Initial program		"New" programs					
		-0.15 change in backfat		+0.30 change in backfat		-0.30 change in backfat	
Z_o	$\sum_{j*} x_{j*}$	Z'	$\sum_{j*} x_{j*}'$	Z'	$\sum_{j*} x_{j*}'$	Z'	$\sum_{j*} x_{j*}'$
$12,242.198	589.875	$12,719.170	582.231	$11,361.563	461.695	$13,190.333	582.231

Table B.4. Changes of linear program coefficients due to +0.15, -0.15, +0.30, and -0.30 changes in feed efficiency: Model II

j	+0.15 change in feed efficiency dc_j/dt_h	-0.15 change in feed efficiency dc_j/dt_h	+0.30 change in feed efficiency dc_j/dt_h	-0.30 change in feed efficiency dc_j/dt_h
15	$-1.09	$+1.09	$-2.18	$+2.18
16	-1.24	+1.24	-2.48	+2.48
17	-1.39	+1.40	-2.78	+2.79
18	-1.54	+1.55	-3.09	+3.09
19	-1.69	+1.70	-3.38	+3.38
20	-1.02	+1.03	-2.05	2.05
21	-1.17	+1.17	-2.33	2.34
22	-1.31	+1.31	-2.62	2.62
23	-1.45	+1.46	-2.91	2.91
24	-1.59	+1.60	-3.19	3.18

Table B.5. Elements in equation 3.51(a) needed to find the economic value of feed efficiency for a +0.15 change in feed efficiency: Model II

j	x_{jo}	x_j	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
15	0	0	$-1.09	0
16	270.8333	270.8333	-1.24	-395.833
17	0	0	-1.39	0
18	0	0	-1.54	0
19	0	0	-1.69	0
20	325.0000	325.0000	-1.02	-331.500
21	0	0	-1.17	0
22	0	0	-1.31	0
23	0	0	-1.45	0
24	0	0	-1.59	0
Σ		595.8333		-667.333

Table B.6. Elements in equation 4.10 needed to find economic values of feed efficiency for -0.15, +0.30, and -0.30 changes in feed efficiency: Model II

Initial program		"New" programs					
		-0.15 change in FE		+0.30 change in FE		-0.30 change in FE	
Z_o	$\sum_{j*} x_{j*}$	Z'	$\sum_{j*} x_{j*'}$	Z'	$\sum_{j*} x_{j*'}$	Z'	$\sum_{j*} x_{j*'}$
$12,242.198	595.8333	$12,941.044	588.113	$11,020.139	466.358	$13,608.995	588.113

Table B.7a. Changes of linear program coefficients due to +0.15 and -0.15 changes in average daily gain: Model II

		+0.15 change in ADG		-0.15 change in ADG	
j	i	da_{ij}/dt_h	dc_j/dt_h	da_{ij}/dt_h	dc_j/dt_h
15	12 22	-0.04 0	+.08	+0.05 0	-.10
16	12 22	-0.03 0	+.09	+0.05 0	-.09
17	12 22	-0.04 0	+.09	+0.06 0	-.11
18	12 13 22	-0.06 0 0	+.10	0 +0.05 0	-.12
19	12 13 22	-0.02 -0.04 0	+.10	0 +0.06 0	-.12
20	17 18 23	-0.04 0 0	+.08	0 +0.05 0	-.10
21	18 23	-0.03 0	+.09	+0.05 0	-.10
22	18 23	-0.04 0	+.09	+0.06 0	-.10
23	18 19 23	-0.06 0 0	+.09	0 +0.05 0	-.11
24	18 19 23	-0.02 -0.04 0	+.09	0 +0.06 0	-.11

Table B.7b. Changes of linear program coefficients due to +0.30 and -0.30 changes in average daily gain: Model II

j	i	+0.30 change in ADG		-0.30 change in ADG	
		da_{ij}/dt_h	dc_j/dt_h	da_{ij}/dt_h	dc_j/dt_h
	11	-0.07		0	
15	12	0	+.15	+0.11	-.22
	22	0		0	
	11	-0.03		0	
16	12	-0.04	+.16	+0.10	-.24
	22	+0.50		0	
	12	-0.07		+0.06	
17	13	0	+.17	+0.07	-.24
	22	+1.00		0	
	12	-0.10		0	
18	13	0	+.18	+0.12	-.25
	22	+0.50		0	
	12	-0.06		0	
19	13	-0.04	+.19	+0.11	-.27
	14	0		+0.02	
	22	+1.00		0	
	17	-0.07		0	
20	18	0	+.14	+0.11	-.21
	23	+1.00		0	
	17	-0.03		0	
21	18	-0.04	+.15	+0.10	-.22
	19	0		+0.02	
	23	-0.05		0	
	18	-0.07		+0.06	
22	19	0	+.16	+0.07	-.23
	23	-0.05		0	
	18	-0.10		0	
23	19	0	+.17	+0.12	-.24
	23	-1.0		0	
	18	-0.06		0	
24	19	-0.04	+.18	+0.12	-.26
	23	-1.0		0	

Table B.8. Elements in equation 3.51 needed to find the economic values of average daily gain for a +0.15 change in average daily gain: Model II

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
15	12 22	0	0	0 1.7357	-0.04 0	0 0	+.08	0
16	12 22	270.8333	270.8333	0 1.7357	-0.03 0	0 0	+.09	+24.375
17	12 22	0	0	0 1.7357	-0.04 0	0 0	+.09	0
18	12 13 22	0	0	0 0 1.7357	-0.06 0 0	0 0 0	+.10	0
19	12 13 22	0	0	0 0 1.7357	-0.02 -0.04 0	0 0 0	+.10	0
20	17 18 23	325.000	325.000	0 0 0.1155	-0.04 0 0	0 0 0	+.08	+26.000
21	18 23	0	0	0 0.1155	-0.03 0	0 0	+.09	0
22	18 23	0	0	0 0.1155	-0.04 0	0 0	+.09	0

Table B.8. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
	18			0	-0.06	0		
23	19	0	0	0	0	0	+.09	0
	23			0.1155	0	0		
	18			0	-0.02	0		
24	19	0	0	0	-0.04	0	+.09	0
	23			0.1155	0	0		
Σ			595.8333			0		+50.375

Table B.9. Elements in equation 3.51 needed to find the economic values of average daily gain for a -0.15 change in average daily gain: Model II

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
15	12	0	0	0	+0.05	0	-.10	0
	22			1.7357	0	0		
16	12	270.8333	270.8333	0	+0.05	0	-.09	-24.375
	22			1.7357	0	0		
17	12	0	0	0	+0.06	0	-.11	0
	22			1.7357	0	0		
18	12	0	0	0	0	0	-.12	0
	13			0	+0.05	0		
	22			1.7357	0	0		
19	12	0	0	0	0	0	-.12	0
	13			0	+0.06	0		
	22			1.7357	0	0		
20	17	325.000	325.000	0	0	0	-.10	-32.500
	18			0	+0.05	0		
	23			0.1155	0	0		
21	18	0	0	0	+0.05	0	-.10	0
	23			0.1155	0	0		
22	18	0	0	0	+0.06	0	-.10	0
	23			0.1155	0	0		

Table B.9. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
	18			0	0	0		
23	19	0	0	0	+0.05	0	-.11	0
	23			0.1155	0	0		
	18			0	0	0		
24	19	0	0	0	+0.06	0	-.11	0
	23			0.1155	0	0		
Σ			595.8333			0		-56.875

Table B.10. Elements in equation 3.51 needed to find the economic values of average daily gain for a -0.30 change in average daily gain: Model II

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
15	12	0	0	0	+0.11	0	-.22	0
	22			1.7357	0	0		
16	12	270.8333	270.8333	0	+0.10	0	-.24	-65.000
	22			1.7357	0	0		
17	12	0	0	0	+0.06	0	-.24	0
	13			0	+0.07	0		
	22			1.7357	0	0		
18	13	0	0	0	+0.12	0	-.25	0
	22			1.7357	0	0		
19	13	0	0	0	+0.11	0	-.27	0
	14			0	+0.02	0		
	22			1.7357	0	0		
20	18	325.000	325.000	0	+0.11	0	-.21	-68.250
	23			0.1155	0	0		
21	18	0	0	0	+0.10	0	-.22	0
	19			0	+0.02	0		
	23			0.1155	0	0		
22	18	0	0	0	+0.06	0	-.23	0
	19			0	+0.07	0		
	23			0.1155	0	0		

Table B.10. Continued

j	i	x_{jo}	x_j	$(z_{n+i}-c_{n+i})$	da_{ij}/dt_h	$(z_{n+i}-c_{n+i})x_{jo}\ da_{ij}/dt_h$	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$
23	19	0	0	0	+0.12	0	-.24	0
	23			0.1155	0	0		
24	19	0	0	0	+0.12	0	-.26	0
	23			0.1155	0	0		
Σ			595.8333			0		-133.250

Table B.11. Elements in equation 4.10 needed to find an economic value for average daily gain for a +0.30 change in average daily gain: Model II

Initial program		"New" program +0.30 change in ADG	
Z_o	$\sum_{j*} x_j{*}$	Z'	$\sum_{j*} x_{j*}'$
$12,242.198	595.833	$12,073.967	555.455

XII. APPENDIX C

Table C.1. Elements in equation 6.2 needed to find the premiums of breeding animals that cause -0.15 changes in backfat: revised Model II

j	x_{jo}	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	$x_{j*'}$ Female	$x_{j*'}$ Male
25	0	\$+.43	0		
26	169.3897	+.42	\$+71.144		
27	0	+.43	0		
28	0	+.42	0		
29	238.4806	+.43	+102.547		
30	0	+.41	0		
31	387.1395	+.40	+154.856		
32	0	+.40	0		
33	0	+.39	0		
34	0	+.40	0		
Σ			\$+328.547	27.6316	1.0

Table C.2. Elements in equation 6.3 needed to find the premiums of breeding animals that cause +0.15 changes in backfat: revised Model II

Initial program			"New" program +0.15 change in backfat		
	$\sum_{j*'} x_{j*'}$			$\sum_{j*'} x_{j*'}$	
Z_o	Female	Male	Z'	Female	Male
\$10,322.992	27.6316	1.0	\$10,010.388	27.6316	1.0

Table C.3. Elements in equation 6.2 needed to find the premiums of breeding animals that cause -0.15 changes in feed efficiency: revised Model II

j	x_{jo}	dc_j/dt_h	$x_{jo}\ dc_j/dt_h$	$x_{j*'}$ Female	$x_{j*'}$ Male
15	0	$+1.09	0		
16	85.5503	+1.24	$+106.082		
17	0	+1.40	0		
18	0	+1.55	0		
19	92.6497	+1.70	+157.504		
20	0	+1.03	0		
21	195.5250	+1.17	+228.765		
22	0	+1.31	0		
23	0	+1.46	0		
24	0	+1.60	0		
Σ			$+492.351	27.6316	1.0

Table C.4. Elements in equation 6.3 needed to find the premiums of breeding animals that cause +0.15 changes in feed efficiency: revised Model II

Initial program			"New" program +0.15 change in FE		
Z_o	$\sum_{j*'} x_{j*'}$ Female	$\sum_{j*'} x_{j*'}$ Male	Z'	$\sum_{j*'} x_{j*'}$ Female	$\sum_{j*'} x_{j*'}$ Male
$10,322.992	27.6316	1.0	$9,857.178	27.6316	1.0

Table C.5. Elements in equation 6.3 needed to find the premiums of breeding animals that cause +0.15 and -0.15 changes in average daily gain

Initial program			"New" programs					
			+0.15 change in ADG			-0.15 change in ADG		
Z_o	$\sum_{j^{*\prime}} x_{j^{*\prime}}$		Z'	$\sum_{j^{*\prime}} x_{j^{*\prime}}$		Z'	$\sum_{j^{*\prime}} x_{j^{*\prime}}$	
	Female	Male		Female	Male		Female	Male
$10,322.992	27.6316	1.0	$10,406.336	27.6316	1.0	$10,242.220	27.6316	1.0

Table C.6. Elements in equation 6.3 needed to find the premiums of breeding animals of lesser and greater breeding potential

			"New" programs					
			Lesser breeding potential			Greater breeding potential		
Initial program			+0.15 changes in backfat and feed efficiency with a -0.15 change in average daily gain			-0.15 changes in backfat and feed efficiency with a +0.15 change in average daily gain		
	$\Sigma_{j*'} x_{j*'}$			$\Sigma_{j*'} x_{j*'}$			$\Sigma_{j*'} x_{j*'}$	
Z_o	Female	Male	Z'	Female	Male	Z'	Female	Male
$10,322.992	27.6316	1.0	$9,486.238	27.6316	1.0	$11,248.404	27.6316	1.0